GradePeak
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# PHYS 4

## CONCEPT PACKET
## Complete

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## Electromagnetic Fields and Light as a Wave

### Electromagnetic Fields

Faradays Law and Maxwells Law describe the mutual induction of electric fields and magnetic fields that produce light

$$Faradays\ Law\ \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \rightarrow \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \qquad Maxwells\ Law\ \oint \vec{B}\cdot d\vec{s} = \mu_0\,\varepsilon_0 \frac{d\Phi_E}{dt} \rightarrow -\frac{\partial B}{\partial x} = \mu_0\,\varepsilon_0 \frac{\partial E}{\partial t}$$

$$Permittivity\ of\ a\ vacuum\ \ \varepsilon_0 = 8.854\times 10^{-12}\,\frac{F}{m} \qquad Permeability\ of\ a\ vacuum\ \ \mu_0 = 4\pi\times 10^{-7}\,\frac{H}{m}$$

The solution of the differential equation pair is an in phase (simultaneous amplitudes) orthogonal electromagnetic field

$$Electric\ Field\ \ E = E_m\sin(kx-\omega t) \quad orthogonal\ to\ and\ in\ phase\ with \quad Magnetic\ Field\ \ B = B_m\sin(kx-\omega t)$$

$$Amplitude\ Relation\ \ c = \frac{E_m}{B_m}\ \ where\ \ c = 3\times 10^8\,\frac{m}{s} \qquad Root\ Mean\ Square\ Relations\ \ E_{rms} = \frac{E_m}{\sqrt{2}}\quad B_{rms} = \frac{B_m}{\sqrt{2}}$$

$$Electric\ Field\ Energy\ Density\ \ u_E = \frac{\varepsilon_0\,E^2}{2} \qquad Magnetic\ Field\ Energy\ Density\ \ u_B = \frac{B^2}{2\mu_0} \qquad For\ light\ u_E = u_B$$

Light is a wave with speed $c$ as the rate at which all points of the wave propagate through space, with wavelength $\lambda$ as the distance between equivalent identical points on the wave, with period $T$ as the time needed for one full wavelength or one full cycle to pass, and frequency $f$ as the number of wavelengths or cycles that pass per second.

$$c = \lambda f \qquad T = \frac{1}{f} = \frac{\lambda}{c} \qquad where\ speed\ of\ light\ wave\ in\ a\ vacuum\ is \quad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 2.998\times 10^8\,\frac{m}{s} \approx 3\times 10^8\,\frac{m}{s}$$

**Electromagnetic Spectrum of Light** is a continuous spectrum of different wavelengths, frequencies, and energies:

$\lambda =$ 1 $cm$ 1 $mm$ (700 650 600 550 500 450 425 400)$nm$ 1 $nm$ 1$pm$

*Radio Microwave Infrared Red Orange Yellow Green Blue Indigo Violet Ultraviolet XRay Gamma Ray*

→→→→→→→→→→→→→ *Wavelength decreases, Frequency increases, Energy Increases* →→→→→→→→→→→→→

The wavelengths of visible light are colored and can be memorized in order of increasing frequency by *ROY G BIV*

The wavelengths longer than visible light are harmless and are used for radio broadcasts and telecommunications.

The wavelengths shorter than visible light are harmful and are only used in small doses for medical and dental imaging.

### Poynting Vector and Electromagnetic Energy Transport

The Poynting Vector has a magnitude of the rate of energy transport per area and is directed with the propagation of the electromagnetic wave which is mutually orthogonal to both the electric field and the magnetic field as follows

$$Poynting\ Vector\ \ \vec{S} = \frac{1}{\mu_0}\,\vec{E}\times\vec{B} \qquad Poynting\ Magnitude\ \ S = \frac{energy\ rate}{area} = \frac{power}{area} = \frac{E\,B}{\mu_0} = \frac{E^2}{c\,\mu_0} = \frac{c\,B^2}{\mu_0}$$

### Light Source Power, Light Intensity, Light Detector Power

Light Source Power is the rate at which energy is emitted from the light source per unit time. The light emanates outward usually in either a spherical shell wave front or a hemispherical shell wave front and has a received intensity

$$Light\ Intensity\ \ I_{received} = \frac{P_{source}}{A_{wavefront\ area}} = S_{avg} = \frac{E_{rms}^2}{c\,\mu_0} = \frac{E_m^2}{2\,c\,\mu_0} = \frac{c\,B_m^2}{2\,\mu_0}\ \ where\ A\ may\ depend\ on\ distance$$

$$A = 4\pi r^2\ \ isotropic\ spherical\ wavefront\ at\ distance\ r \qquad A = 2\pi r^2\ \ hemispherical\ wavefront\ at\ distance\ r$$

$$Light\ Detector\ Power \qquad P_{received} = \frac{P_{source}\,A_{receiver}}{A_{wavefront\ area}} \qquad where\ A_{receiver}\ is\ the\ receiver\ cross\ sectional\ area$$

*Huygens Principle states that each point on a light wavefront acts as a point source for the next wavefront*

### Change in Momentum and Radiation Pressure of Light on a surface

Radiation Pressure is the force per area created as light encounters a surface which causes a change in its momentum.

$$Light\ Completely\ Absorbed \quad Change\ in\ Momentum\ \ \Delta p = \frac{\Delta U}{c} \quad Force\ \ F = \frac{I\,A}{c} \quad Radiation\ Pressure\ \ p_r = \frac{I}{c}$$

$$Light\ Completely\ Reflected\ Change\ in\ Momentum\ \ \Delta p = \frac{2\,\Delta U}{c} \quad Force\ \ F = \frac{2\,I\,A}{c} \quad Radiation\ Pressure\ \ p_r = \frac{2\,I}{c}$$

Where $\Delta U$ is the change in the energy of light, $I$ is the light intensity, $A$ is the receiver area, and $c$ is the speed of light.

## Polarization of Light, Reflection and Refraction of Light

Polarization, Reflection, and Refraction are different interactions that may occur between light waves and a surface.

### Polarization of Light

Electromagnetic Field waves normally occur in random oriented planes orthogonal to the wave propagation. Polarization of Light occurs when Field waves are permitted to transmit their components only along one axis, the Polarization Axis.

$I$ *Final Intensity* $I_0$ *Initial Intensity* $\theta_{polarizer\ a\ and\ polarizer\ b}$ *Relative Angle between polarizers a and b*

$P$ *Final Power* $P_0$ *Initial Power* $\theta_{light\ and\ polarizer\ a}$ *Relative Angle between polarized light and polarizer a*

*Unpolarized Light through One Polarizer* $I = \frac{1}{2} I_0$ $P = \frac{1}{2} P_0$

*Unpolarized Light through Two Polarizers* $I = \frac{1}{2} I_0 \cos^2 \theta_{polarizer\ 1\ and\ polarizer\ 2}$

$$P = \frac{1}{2} P_0 \cos^2 \theta_{polarizer\ 1\ and\ polarizer\ 2}$$

*Unpolarized Light through Three Polarizers* $I = \frac{1}{2} I_0 \cos^2 \theta_{polarizer\ 1\ and\ polarizer\ 2} \cos^2 \theta_{polarizer\ 2\ and\ polarizer\ 3}$

$$P = \frac{1}{2} P_0 \cos^2 \theta_{polarizer\ 1\ and\ polarizer\ 2} \cos^2 \theta_{polarizer\ 2\ and\ polarizer\ 3}$$

*Polarized Light through One Polarizer* $I = I_0 \cos^2 \theta_{polarization\ axis\ and\ polarizer\ 1}$

$$P = P_0 \cos^2 \theta_{polarization\ axis\ and\ polarizer\ 1}$$

*Polarized Light through Two Polarizers* $I = I_0 \cos^2 \theta_{polarization\ axis\ and\ polarizer\ 1} \cos^2 \theta_{polarizer\ 1\ and\ polarizer\ 2}$

$$P = P_0 \cos^2 \theta_{polarization\ axis\ and\ polarizer\ 1} \cos^2 \theta_{polarizer\ 1\ and\ polarizer\ 2}$$

### Reflection and Refraction of Light

When light encounters an object surface, part of it rebounds as reflection and part of it transmits through as refraction.

*Speed of Light, Wavelength, Frequency in medium with index of refraction n* $v_n = \frac{c}{n}$ $\lambda_n = \frac{\lambda}{n}$ $f_n = f$

*All Angles θ for reflection and refraction are always measured to the normal line of the surface only!*

**Reflection**

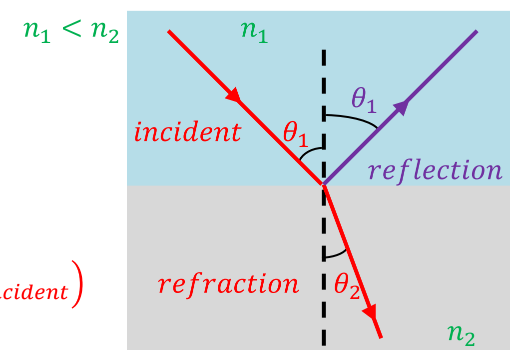
$\theta_1 = \theta_2$

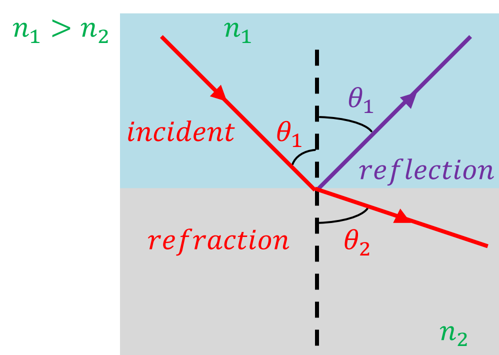
$\theta_{reflection} = \theta_{incident}$

**Refraction**

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

$\theta_{refraction} = \sin^{-1}\left(\frac{n_1}{n_2} \sin \theta_{incident}\right)$




**Perceived Depth due to Refraction**

The Perceived Depth of an object in a fluid for an observer directly above the object is caused by a difference in media.

*Perceived Depth for vertical observer to an object in a fluid* $\frac{depth_{perceived\ for\ object}}{n_{medium\ for\ observer}} = \frac{depth_{actual\ for\ object}}{n_{fluid\ containing\ object}}$

**Critical Angle for Total Internal Reflection and No Refraction**

The Critical Angle is the exact incident angle at which refracted light transmits along the surface of the object. Any light with incident angle greater than the Critical Angle will have refracted angle greater than 90° and will internally reflect.

*Critical Incident Angle for Total Internal Reflection* $\theta_c = \theta_{incident} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ *will cause* $\theta_{refracted} = 90°$

*Total Internal Reflection is only possible for* $n_1 > n_2$ *and will occur for all light with* $\theta_{incident} > \theta_c$

**Brewsters Angle for Complete Polarization of Reflected Light**

Reflected Light is always partially polarized but will be completely polarized if the incident angle is Brewsters Angle

*Brewters Angle for Complete Polarization of Reflected Light* $\theta_B = \theta_{incident} = \theta_{reflected} = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

## Images and Optics

Images are formed when light rays Converge to a Real Image or Diverge to a back Virtual Image. Optics includes objects which may produce images through reflection such as mirrors and produce images through refraction such as lenses.

$Diverging\ Mirror\ or\ Diverging\ Lens\ always\ causes\ divergent\ light\ rays\ and\ will\ only\ form\ Virtual\ Images$

$Converging\ Mirror\ or\ Converging\ Lens\ will\ cause\ convergent\ light\ rays\ and\ will\ form\ Real\ Images\ unless$
$object\ is\ placed\ within\ its\ focal\ length\ then\ it\ will\ cause\ divergent\ light\ rays\ and\ will\ form\ Virtual\ Images$

$h_o = height\ of\ object \quad + for\ physical\ object\ or\ upright\ image\ as\ object \quad - for\ inverted\ image\ as\ object$

$h_i = height\ of\ image \quad + for\ upright\ image\ relative\ to\ object \quad - for\ inverted\ image\ relative\ to\ object$

$p = (1-m)f = distance\ of\ object\ from\ mirror\ or\ lens \quad + for\ physical\ object \quad - for\ virtual\ image\ as\ object$

$i = \left(1 - \frac{1}{m}\right)f = distance\ of\ image\ from\ mirror\ or\ lens \quad + for\ real\ image \quad - for\ virtual\ image$

$r = curvature\ radius\ of\ mirror\ or\ lens \ + for\ converging\ object \ - for\ diverging\ object \ \infty\ for\ plane\ mirror$

$f = \frac{r}{2} = focal\ length\ of\ mirror\ or\ lens \ + for\ converging\ object \ - for\ diverging\ object \ \infty\ for\ plane\ mirror$

$m = \frac{h_i}{h_o} = -\frac{i}{p} = magnification\ of\ mirror\ or\ lens \quad + for\ upright\ image \quad - for\ inverted\ image$

$NP = near\ point\ of\ the\ observers\ eye\ or\ closest\ object\ in\ focus \quad NP = 25\ cm \quad for\ a\ normal\ human\ eye$

$FP = far\ point\ of\ the\ observers\ eye\ or\ furthest\ object\ in\ focus \quad FP = \infty \quad for\ a\ normal\ human\ eye$

### Images formed by Mirror Equations

Mirrors form images by reflection of light which can either converge into a Real Image or diverge into a Virtual Image.

$$\frac{1}{i} + \frac{1}{p} = \frac{1}{f} \qquad m = \frac{h_i}{h_o} = -\frac{i}{p} \qquad f = \frac{r}{2} \qquad i = (1-m)f \qquad p = \left(1 - \frac{1}{m}\right)f \qquad m = \frac{f}{f-p}$$

$$For\ a\ Plane\ Mirror \quad f = \infty \quad so\ that \quad i = -p \quad m = 1$$

### Images formed by Lens Equations

Lenses form images by reflection of light which can either converge into a Real Image or diverge into a Virtual Image.

$$One\ Lens\ System \quad \frac{1}{i} + \frac{1}{p} = \frac{1}{f} \qquad m = \frac{h_i}{h_o} = -\frac{i}{p} \qquad f = \frac{r}{2} \qquad i = (1-m)f \qquad p = \left(1 - \frac{1}{m}\right)f \qquad m = \frac{f}{f-p}$$

$$Two\ Lens\ System \quad \frac{1}{i_1} + \frac{1}{p_1} = \frac{1}{f_1} \qquad i_1 + p_2 = d = lens\ separation \qquad \frac{1}{i_2} + \frac{1}{p_2} = \frac{1}{f_2} \qquad f_1 = \frac{r_1}{2} \qquad f_2 = \frac{r_2}{2}$$

$$Two\ Lens\ Overall\ Magnification \quad m = \frac{h_{i2}}{h_{o1}} = m_1\, m_2 = \left(\frac{i_1}{p_1}\right)\left(\frac{i_2}{p_2}\right) \quad where \quad m_1 = \frac{h_{i1}}{h_{o1}} = -\frac{i_1}{p_1} \quad m_2 = \frac{h_{i2}}{h_{o2}} = -\frac{i_2}{p_2}$$

$$Refracting\ Surface\ Equation \quad \frac{n_1}{p} + \frac{n_2}{i} = \frac{(n_2 - n_1)}{r} \qquad Lens\ Makers\ Equation \quad \frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

**Magnifying Glass** is a single converging lens whose purpose is to increase the angular size of an object to be viewed

$$With\ a\ Magnifying\ Glass \quad m = \frac{\theta_{image}}{\theta_{object}} = \frac{near\ point\ of\ eye}{lens\ focal\ length} = \frac{NP}{f} \qquad \theta_{image} = \frac{h_o}{f} = \frac{m\, h_o}{d_o} \qquad \theta_{object} = \frac{h_o}{NP}$$

$$Without\ a\ Magnifying\ Glass \quad m = \frac{\theta_{image}}{\theta_{object}} = 1 \qquad \theta_{image} = \theta_{object} = \frac{height\ of\ object}{near\ point\ of\ eye} = \frac{h_o}{NP}$$

**Eye Glasses** are either converging lenses to correct for Farsightedness or diverging lenses to correct for Nearsightedness

$$For\ Farsightedness \quad i = -near\ point\ of\ eye = -NP \quad and \quad p = 25\ cm \quad so\ that \quad -\frac{1}{NP} + \frac{1}{25\ cm} = \frac{1}{f}$$

$$For\ Nearsightedness \quad i = -far\ point\ of\ eye = -FP \quad and \quad p = \infty \quad so\ that \quad -\frac{1}{FP} = \frac{1}{f} \quad or \quad f = -FP$$

**Refracting Telescope** is a two lens system that greatly magnifies image height from object height for very distant objects

$$m = \frac{\theta_i}{\theta_o} = \frac{\theta_{eyepiece\ lens}}{\theta_{objective\ lens}} = -\frac{f_{objective\ lens}}{f_{eyepiece\ lens}} \qquad \theta_{objective\ lens} = \frac{h_0}{d_0} = \frac{h_{from\ objective}}{f_{objective\ lens}} \qquad \theta_{eyepiece\ lens} = \frac{h_{from\ objective}}{f_{eyepiece\ lens}}$$

**Compound Microscope** is a two lens system that greatly magnifies image height from object height for very near objects

$$M = \frac{\theta_i}{\theta_o} = m_{objective}\, m_{eyepiece} = -\left(\frac{tube\ length}{f_{objective\ lens}}\right)\left(\frac{near\ point\ of\ eye}{f_{eyepiece\ lens}}\right) = -\left(\frac{s}{f_{objective\ lens}}\right)\left(\frac{NP}{f_{eyepiece\ lens}}\right)$$

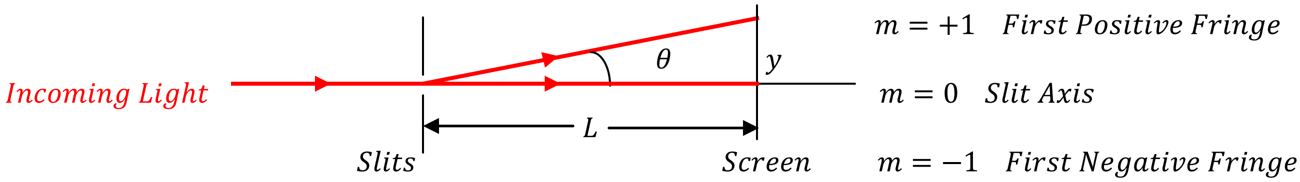
| Type of Mirror | Sign of Focal Length f | Location of Object p | Sign of Image Distance i | Type of Image | Sign of Magnification m | Value of Magnification m | Orientation and Size of Image |
|---|---|---|---|---|---|---|---|
| Converging Concave | *Positive* | $0 < p < f$ | *Negative* | *Virtual* | *Positive* | $\|m\| > 1$ | *Upright Enlarged* |
| Converging Concave | *Positive* | $p = f$ | *No Image* | *No Image* | *No Image* | *No Image* | *No Image* |
| Converging Concave | *Positive* | $f < p < 2f$ | *Positive* | *Real* | *Negative* | $\|m\| > 1$ | *Inverted Enlarged* |
| Converging Concave | *Positive* | $p = 2f$ | *Positive* | *Real* | *Negative* | $\|m\| = 1$ | *Inverted Same Size* |
| Converging Concave | *Positive* | $p > 2f$ | *Positive* | *Real* | *Negative* | $\|m\| < 1$ | *Inverted Reduced* |
| Parallel Plane | *Infinity* | *Anywhere* | *Negative* $i = -p$ | *Virtual* | *Positive* | $\|m\| = 1$ | *Upright Same Size* |
| Diverging Convex | *Negative* | $0 < p < f$ | *Negative* | *Virtual* | *Positive* | $\|m\| < 1$ | *Upright Reduced* |
| Diverging Convex | *Negative* | $p = f$ | *Negative* | *Virtual* | *Positive* | $\|m\| = +\frac{1}{2}$ | *Upright Reduced* |
| Diverging Convex | *Negative* | $f < p < 2f$ | *Negative* | *Virtual* | *Positive* | $\|m\| < 1$ | *Upright Reduced* |
| Diverging Convex | *Negative* | $p = 2f$ | *Negative* | *Virtual* | *Positive* | $\|m\| = +\frac{1}{3}$ | *Upright Reduced* |
| Diverging Convex | *Negative* | $p > 2f$ | *Negative* | *Virtual* | *Positive* | $\|m\| < 1$ | *Upright Reduced* |
| Type of Lens | Sign of Focal Length f | Location of Object p | Sign of Image Distance i | Type of Image | Sign of Magnification m | Value of Magnification m | Orientation and Size of Image |
| Converging Convex | *Positive* | $0 < p < f$ | *Negative* | *Virtual* | *Positive* | $\|m\| > 1$ | *Upright Enlarged* |
| Converging Convex | *Positive* | $p = f$ | *No Image* | *No Image* | *No Image* | *No Image* | *No Image* |
| Converging Convex | *Positive* | $f < p < 2f$ | *Positive* | *Real* | *Negative* | $\|m\| > 1$ | *Inverted Enlarged* |
| Converging Convex | *Positive* | $p = 2f$ | *Positive* | *Real* | *Negative* | $\|m\| = 1$ | *Inverted Same Size* |
| Converging Convex | *Positive* | $p > 2f$ | *Positive* | *Real* | *Negative* | $\|m\| < 1$ | *Inverted Reduced* |
| Diverging Concave | *Negative* | $0 < p < f$ | *Negative* | *Virtual* | *Positive* | $\|m\| < 1$ | *Upright Reduced* |
| Diverging Concave | *Negative* | $p = f$ | *Negative* | *Virtual* | *Positive* | $\|m\| = +\frac{1}{2}$ | *Upright Reduced* |
| Diverging Concave | *Negative* | $f < p < 2f$ | *Negative* | *Virtual* | *Positive* | $\|m\| < 1$ | *Upright Reduced* |
| Diverging Concave | *Negative* | $p = 2f$ | *Negative* | *Virtual* | *Positive* | $\|m\| = +\frac{1}{3}$ | *Upright Reduced* |
| Diverging Concave | *Negative* | $p > 2f$ | *Negative* | *Virtual* | *Positive* | $\|m\| < 1$ | *Upright Reduced* |

## Interference of Light

Interference of Light occurs when separate wave fronts of light interact and cause either Constructive Interference with increased intensity bright regions or Destructive Interference with decreased intensity dark regions by combined waves

$$y_1 + y_2 = A_1 \sin(kx - \omega t) + A_2 \sin(kx - \omega t + \phi) = (A_1 + A_2 \cos\phi)\sin(kx - \omega t) + (A_2 \sin\phi)\cos(kx - \omega t)$$

$Amplitude\ \ |y_1 + y_2| = \sqrt{A_1^2 + A_2^2 + A_1\,A_2 \cos\phi}$ $\quad Percent\ Construct = \sin^2\left(\frac{\phi}{2}\right)$ $\quad Percent\ Destruct = \sin^2\left(\frac{\phi}{2}\right)$




$m = integer\ with\ negative\ on\ one\ side\ and\ positive\ on\ other\ side\ of\ the\ slit\ axis = \cdots, -3, -2, -1, 0, 1, 2, 3, \ldots$

$a = slit\ width \quad d = slit\ separation \quad \theta = angular\ distance\ of\ fringe\ from\ slit\ axis\ or\ angular\ size\ of\ object$

$L = distance\ of\ screen\ or\ object \quad y = linear\ distance\ of\ fringe\ from\ slit\ axis\ or\ linear\ size\ of\ object$

$Speed\ of\ Light, Wavelength, Frequency\ in\ medium\ with\ index\ of\ refraction\ n \quad v_n = \frac{c}{n} \quad \lambda_n = \frac{\lambda}{n} \quad f_n = f$

### Single Slit Diffraction Interference

Single Slit Diffraction Interference occurs as light passes through slit width $a$ on the order of the light wavelength.

$Destructive\ Interference\ Dark\ Minima\ Locations \quad a \sin\theta = m\frac{\lambda}{n} \quad a\,\frac{y}{L} = m\,\frac{\lambda}{n} \quad since \quad \sin\theta = \frac{y}{L}$

$Width\ of\ Central\ Bright\ Maximum\ is\ the\ distance\ between\ first\ bright\ fringes\ on\ each\ side \quad Width = \frac{2\,L\,\lambda}{a\ \ n}$

$Constructive\ Interference\ Bright\ Maxima\ Locations \quad a \sin\theta \lesssim \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad a\,\frac{y}{L} = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad since\ \sin\theta = \frac{y}{L}$

$Intensity\ at\ Angle\ Location\ \ I(\theta) = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \alpha = \frac{\pi a}{\lambda}\sin\theta \quad Destructive\ Interference\ Minima\ if\ \ \alpha = m\,\pi$

### Double Slit Diffraction Interference

Double Slit Diffraction Interference occurs as light passes through slit separation $d$ on the order of the light wavelength.

$Constructive\ Interference\ Bright\ Maxima\ Locations \quad d \sin\theta = m\frac{\lambda}{n} \quad d\,\frac{y}{L} = m\frac{\lambda}{n} \quad since \quad \sin\theta = \frac{y}{L}$

$Missing\ Bright\ Fringes\ from\ slit\ width\ occur\ at\ integer\ multiples\ of\ \frac{d}{a} \quad Missing\ Bright\ Fringes = \pm\frac{k\,d}{a}$

$Destructive\ Interference\ Dark\ Minima\ Locations \quad d \sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad d\,\frac{y}{L} = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad since\ \ \sin\theta = \frac{y}{L}$

$Intensity\ at\ Angle\ Location\ \ I(\theta) = I_0(\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \alpha = \frac{\pi a}{\lambda}\sin\theta \quad \beta = \frac{\pi d}{\lambda}\sin\theta$

$Constructive\ Interference\ Maxima\ if \quad \alpha = m\left(\frac{a}{d}\right)\pi \quad and \quad \beta = m\,\pi$

$Destructive\ Interference\ Minima\ if \quad \alpha = \left(m + \frac{1}{2}\right)\left(\frac{a}{d}\right)\pi \quad or \quad \beta = \left(m + \frac{1}{2}\right)\pi$

### Multiple Slit Diffraction Grating Interference

Double Slit Diffraction Interference occurs as light passes through slit separations $d$ on the order of the light wavelength.

$Constructive\ Interference\ Bright\ Maxima\ Locations \quad d \sin\theta = m\frac{\lambda}{n} \quad d\,\frac{y}{L} = m\frac{\lambda}{n} \quad since \quad \sin\theta = \frac{y}{L}$

$Destructive\ Interference\ Dark\ Minima\ Locations \quad d \sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad d\,\frac{y}{L} = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad since\ \ \sin\theta = \frac{y}{L}$

$Slit\ Separation \quad d = \frac{1}{number\ of\ slits\ per\ unit\ length} = \frac{1}{n} \quad where \quad n = \frac{number\ of\ slits}{grating\ length} = \frac{N}{l}$

$Half\ Width\ at\ Angle\ \theta \ \ \Delta\theta_{hw} = \frac{\lambda}{N\,d\cos\theta} \quad Dispersion\ \ D = \frac{\lambda_{avg}}{\Delta\lambda} = \frac{m}{d\cos\theta} \quad Resolving\ Power\ \ R = N\,m$

## Circular Slit Diffraction Interference and Raleighs Criterion of Resolution

Circular Slit Diffraction Interference occurs as light passes through slit diameter $d$ on the order of the light wavelength.

$Constructive\ Interference\ Bright\ Maxima\ Locations \quad d \sin\theta = 1.22\frac{\lambda}{n} \quad d\ \frac{y}{L} = 1.22\frac{\lambda}{n} \quad d\ \theta_{radians} = 1.22\frac{\lambda}{n}$

$Diameter\ of\ Central\ Bright\ Maximum\ is\ double\ the\ radius\ of\ the\ first\ dark\ fringe \quad Diameter = \frac{2.44}{d}\left(\frac{\lambda}{n}\right)$

$Raleighs\ Criterion\ of\ Resolution\ for\ two\ sources\ requires\ the\ separation\ be\ no\ less\ than\ the\ central\ radius$

$Raleighs\ Criterion\ for\ resolution\ of\ light\ sources \quad d\ \theta_{radians\ resolution\ angle} = 1.22\frac{\lambda}{n} \quad d\ \frac{y}{L} = 1.22\frac{\lambda}{n}$

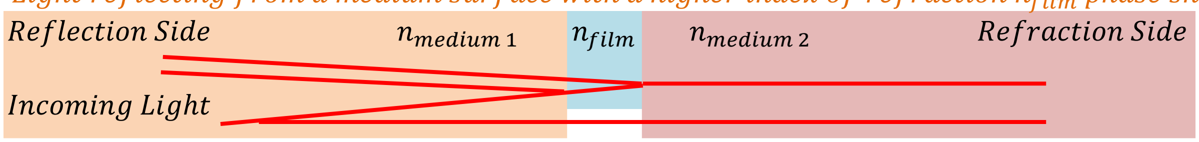
## Thin Film Interference

Thin Film Diffraction occurs as light passes through a thin medium with width $t$ on the order of the light wavelength. The reflection is on the same side of thin film and the refraction is the opposite side of thin film that light originated from.

$Light\ refracting\ through\ a\ medium\ surface\ with\ any\ index\ of\ refraction\ n_{film}\ does\ not\ phase\ shift\ at\ all.$

$Light\ reflecting\ from\ a\ medium\ surface\ with\ a\ lower\ index\ of\ refraction\ n\ does\ not\ phase\ shift\ at\ all.$

$Light\ reflecting\ from\ a\ medium\ surface\ with\ a\ higher\ index\ of\ refraction\ n_{film}\ phase\ shifts\ by\ \pi\ radians.$




**Reflection from thin film with index of refraction between indices of refraction for media on each side.**

$n_{film}\ between\ both\ n_1\ and\ n_3 \quad Constructive\ Interference\ Bright\ Maxima \quad 2\ t = m\frac{\lambda}{n_{film}}$

$n_{film}\ between\ both\ n_1\ and\ n_3 \quad Destructive\ Interference\ Dark\ Minima \quad 2\ t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$

**Reflection from thin film with index of refraction greater than or less that indices of refraction for media on each side.**

$n_{film}\ greater\ or\ lesser\ than\ both\ n_1\ and\ n_3 \quad Constructive\ Interference\ Bright\ Maxima \quad 2\ t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$

$n_{film}\ greater\ or\ lesser\ than\ both\ n_1\ and\ n_3 \quad Destructive\ Interference\ Dark\ Minima \quad 2\ t = m\frac{\lambda}{n_{film}}$

**Refraction through thin film with index of refraction of any relation to indices of refraction for media on each side.**

$Refraction \quad Phase\ Shift \quad \Delta\phi = \frac{2\ \pi\ L(n_2 - n_1)}{\lambda} \quad Bright\ Maxima \quad m = \frac{L(n_2 - n_1)}{\lambda} \quad t = m\frac{\lambda}{n_{film}}$

$Refraction \quad Phase\ Shift \quad \Delta\phi = \frac{2\pi L(n_2 - n_1)}{\lambda} \quad Dark\ Minima \quad m + \frac{1}{2} = \frac{L(n_2 - n_1)}{\lambda} \quad t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{film}}$

## Michelson Interferometer

The Michelson Interferometer splits light into two beams, one which reflects from a fixed mirror and one which reflects from a movable mirror, to measure very short lengths or medium index of refraction by counts of interference fringes $N$.

$different\ path\ lengths\ L_{long\ path}\ and\ L_{short\ path} \quad N_{long\ path} - N_{short\ path} = \frac{2\ n}{\lambda_{source}}\left(L_{long\ path} - L_{short\ path}\right)$

$different\ refraction\ indices\ n_{medium\ 1}\ and\ n_{medium\ 2} \quad N_{medium\ 1} - N_{medium\ 2} = \frac{2\ L}{\lambda_{source}}\left(n_{medium\ 1} - n_{medium\ 2}\right)$

## Atomic Structure X Ray Bragg Diffraction Interference

X Ray Bragg Diffraction Interference occurs as light reflects from atomic layer separation $d$ on order of X ray wavelength.

$Constructive\ Interference\ Bright\ X\ Ray\ Maxima \quad 2\ d \sin\theta = m\frac{\lambda}{n_{substance}}$

$Destructive\ Interference\ Dark\ X\ Ray\ Minima \quad 2\ d \sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_{substance}}$

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## Relativity of Space and Time

Relativity of Space and Time is the relations between kinematic quantities for accelerating reference frames in General Relativity and for inertial or non-accelerating reference frames in Special Relativity. General Relativity is based on the Equivalence Principle and Special Relativity is based on the Relativity Postulate and the Speed of Light Postulate.

*Equivalence Principle of General Relativity* *It is not possible by any experiments within a reference frame to determine whether it is accelerating or under the influence of gravity. Acceleration equals Gravitation.*

*Relativity Postulate of Special Relativity* *The laws of physics are the same for observers in all inertial reference frames. Simultaneous events in one inertial frame are not simultaneous in all inertial frames.*

*Speed of Light Postulate of Special Relativity* *The speed of light in vacuum has the same value c in all directions and in all inertial reference frames. The speed of light c is the ultimate speed in any direction.*

### Relativistic Quantities

An object moving at speeds that approach the speed of light $c$ has kinematic equations becoming more complex in form.

*Speed Parameter* $\beta = \frac{v}{c} \quad 0 \le \beta \le 1$ *Lorentz Factor* $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \quad 1 \le \gamma \le \infty$

*Time Interval Dilation* $\Delta t = \gamma \Delta t_0 = \frac{\Delta t_0}{\sqrt{1-\beta^2}} = \frac{\Delta t_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ *where proper time $\Delta t_0$ is the shortest time interval and is always measured in the frame with the events occuring at the same location.*

*Length Contraction* $L = \frac{L_0}{\gamma} = L_0\sqrt{1-\beta^2} = L_0\sqrt{1-\left(\frac{v}{c}\right)^2}$ *where proper length or rest length $L_0$ is the longest length and is always measured in the frame which is at rest with the object.*

*Momentum* $p = \gamma\, m\, v = \frac{m\, v}{\sqrt{1-\beta^2}} = \frac{m\, v}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$ *Kinetic Energy* $K = m\, c^2\,(\gamma - 1) = m\, c^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right)$

*Rest Mass Energy* $E_0 = m\, c^2$ *Total Energy* $E = E_0 + K = \gamma\, m\, c^2 = \frac{m\, c^2}{\sqrt{1-\beta^2}} = \frac{m\, c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

*Energy Momentum Relations* $(p\, c)^2 = K^2 + 2\, K\, m\, c^2 \qquad E^2 = (p\, c)^2 + E_0^2 = (p\, c)^2 + (m\, c^2)^2$

*Relative Speed Relations* $v_{frame\ 2\ as\ seen\ by\ frame\ 1} = \frac{v_{frame\ 2} - v_{frame\ 1}}{\left(1 - \frac{v_{frame\ 1}\, v_{frame\ 2}}{c^2}\right)} \qquad v_2 = \frac{v_{frame\ 2\ as\ seen\ by\ frame\ 1}}{\left(1 + \frac{v_{frame\ 1}\, v_{frame\ 2}}{c^2}\right)}$

### Lorentz Transormations

The Lorentz Transformations relate the spacetime coordinates of a single event as observed in two inertial frames, with the second frame $S'$ moving at a speed $v$ in the x direction relative to the first frame $S$. The spacetime coordinates are

*x coordinate* $x' = \gamma(x - v\, t)$ *y coordinate* $y' = y$ *z coordinate* $z' = z$ *time coordinate* $t' = \gamma\left(t - \frac{v\, x}{c^2}\right)$

### Doppler Effect for Light

Doppler Effect occurs when a light wave with speed $c$ is either emitted from a moving source $v_{source}$ received by a moving detector $v_{detector}$ or both. The received sound wave cycle frequency $f_{detector}$ and wavelength $\lambda_{detector}$ at the detector is related to the emitted sound wave cycle frequency $f_{source}$ and wavelength $\lambda_{source}$ from the source as

$f_{detector} = f_{source}\sqrt{\left(\frac{1 \pm \beta_{relative}}{1 \mp \beta_{relative}}\right)} \qquad \lambda_{detector} = \lambda_{source}\sqrt{\left(\frac{1 \mp \beta_{relative}}{1 \pm \beta_{relative}}\right)} \qquad \frac{v_{relative}}{c} = \frac{1-\left(\frac{f_{detector}}{f_{source}}\right)^2}{1+\left(\frac{f_{detector}}{f_{source}}\right)^2} = \frac{1-\left(\frac{\lambda_{source}}{\lambda_{detector}}\right)^2}{1+\left(\frac{\lambda_{source}}{\lambda_{detector}}\right)^2}$

*Sign of* $\beta_{relative} = \frac{v_{relative}}{c} = \frac{v_1 - v_2}{c}$ *+ approaching source and detector − receding source and detector*

*Sign of* $\beta_{relative} = \frac{v_{relative}}{c} = \frac{v_1 - v_2}{c}$ *− approaching source and detector + receding source and detector*

## Photons as Particles of Light

Light behaves like a particle with finite energy and momentum in a quantum packet, interacting with mass particles in the Photoelectric Effect or Compton Shift. Energy $E$ and Momentum $p$ carried by light of frequency $f$ and wavelength $\lambda$

$$c = \lambda f \qquad Photon\ Momentum\ \ p = \frac{h}{\lambda} = \frac{h\,f}{c} = \frac{E}{c} \qquad Photon\ Energy\ for\ single\ quanta\ \ E = h\,f = \frac{h\,c}{\lambda}$$

$$Energy\ Difference\ \ \Delta E = h\left(f_{final} - f_{initial}\right) = h\,c\left(\frac{1}{\lambda_{final}} - \frac{1}{\lambda_{initial}}\right) \qquad Energy\ N\ quanta\ \ E = N\,h\,f = \frac{N\,h\,c}{\lambda}$$

$$Plancks\ Constant\ \ \ h = 6.63 \times 10^{-34}\ J \cdot s = 4.14 \times 10^{-15}\ eV \cdot s \qquad Speed\ of\ Light\ \ \ c = 3 \times 10^{8}\,\frac{m}{s}$$

**Electromagnetic Spectrum of Light** is a continuous spectrum of different wavelengths, frequencies, and energies:

$\lambda =$ 1 $cm$ 1 $mm$ (700 650 600 550 500 450 425 400)$nm$ 1 $nm$ 1$pm$

*Radio Microwave Infrared Red Orange Yellow Green Blue Indigo Violet Ultraviolet XRay Gamma Ray*

→→→→→→→→→→→→→ *Wavelength decreases, Frequency increases, Energy Increases* →→→→→→→→→→→→→

The Electromagnetic Energy of Light is determined solely by its Wave Property of Wavelength or Frequency.

The wavelengths of visible light are colored and can be memorized in order of increasing frequency by *ROY G BIV*

The wavelengths longer than visible light are harmless and are used for radio broadcasts and telecommunications.

The wavelengths shorter than visible light are harmful and are only used in small doses for medical and dental imaging.

Photon Intensity and Photon Flux are the photon number count and a measure of the source brightness, but not energy.

$$Photon\ Intensity \qquad I_{received} = \frac{P_{source}}{A_{wavefront\ area}} \qquad where\ A_{wavefront\ area}\ may\ depend\ on\ distance$$

$$A = 4\pi r^2\ \ isotropic\ spherical\ wavefront\ at\ distance\ r \qquad A = 2\pi r^2\ \ hemispherical\ wavefront\ at\ distance\ r$$

$$Photon\ Detector\ Power \qquad P_{received} = \frac{P_{source}\,A_{receiver}}{A_{wavefront\ area}} \qquad where\ A_{receiver}\ is\ the\ receiver\ cross\ sectional\ area$$

$$Photon\ Rate_{source} = \frac{Total\ Power}{Photon\ Energy} = \frac{P}{E} = \frac{I\,A}{E} \quad where\ \ E = h\,f = \frac{h\,c}{\lambda} \quad and \quad Intensity\ \ I = \frac{Power}{A_{receiver}}$$

$$Photon\ Flux \qquad F_{received} = \frac{Photon\ Rate_{source}}{A_{wavefront\ area}} \qquad where\ A_{wavefront\ area}\ may\ depend\ on\ distance$$

$$A = 4\pi r^2\ \ isotropic\ spherical\ wavefront\ at\ distance\ r \qquad A = 2\pi r^2\ \ hemispherical\ wavefront\ at\ distance\ r$$

### Photoelectric Effect

In the Photoelectric Effect, photons collide with a surface ejecting loosely bound valence electrons with binding energy work function $\phi$. The collision causes a loss in photon energy $E$ and frequency $f$ with an increase in wavelength $\lambda$ while imparting up to a maximum kinetic energy $K_{max}$ to the ejected electrons. The energies of the interaction are conserved

$$E_{photon} = K_{max} + \phi \quad with \quad E_{photon} = h\,f = \frac{h\,c}{\lambda} \qquad K_{max} = \frac{1}{2} m_{electron} v_{max}^2 \qquad Stopping\ Potential\ \ e\,V_{stop} = K_{max}$$

$$\lambda_{cutoff} = \frac{h\,c}{\phi} \qquad If\ \lambda > \lambda_{cutoff}\ no\ photoelectric\ effect\ occurs \qquad If\ \lambda < \lambda_{cutoff}\ photoelectric\ effect\ occurs$$

$$m_{electron} = 9.109 \times 10^{-31}\ kg \qquad h = 6.63 \times 10^{-34}\ J \cdot s = 4.14 \times 10^{-15}\ eV \cdot s \qquad c = 3 \times 10^{8}\,\frac{m}{s}$$

### Compton Shift

The Compton Shift is the change in photon energy $E$, frequency $f$, and wavelength $\lambda$ as a function of rebound angle $\phi$ after a collision with a target electron. The photon energy difference is imparted as kinetic energy to the target electron.

$$\Delta\lambda = \lambda_{final} - \lambda_{initial} = \frac{h}{m_{electron}\,c}(1 - \cos\phi) \qquad \frac{1}{f_{final}} - \frac{1}{f_{initial}} = \frac{h}{m_{electron}\,c^2}(1 - \cos\phi)$$

$$\frac{1}{E_{final}} - \frac{1}{E_{initial}} = \frac{1}{m_{electron}\,c^2}(1 - \cos\phi) \qquad K_{electron} = -\left(E_{final} - E_{initial}\right) = \frac{1}{2}\,m_{electron}\,v_{electron}^2$$

$$m_{electron} = 9.109 \times 10^{-31}\ kg \qquad h = 6.63 \times 10^{-34}\ J \cdot s = 4.14 \times 10^{-15}\ eV \cdot s \qquad c = 3 \times 10^{8}\,\frac{m}{s}$$

$$m_{electron}\,c^2 = 8.187 \times 10^{-14}\ J = 5.111 \times 10^{5}\ eV = 511.1\ keV$$

## Matter Waves, Trapped Particles, Quantized Energy

Particle Wave Duality requires mass particles to exhibit behavior attributed to wave properties, including interference or zero probability of detection. A trapped particle will exhibit measurable quantized time independent standing waves.

### Schrodingers Equation

The Schrodinger Equation has wavefunction solutions for particles that are either trapped in a potential well or are free

$$\frac{d^2\,\Psi}{dx^2}+\frac{8\,\pi^2\,m}{h^2}[E-U(x)]\,\Psi=0 \qquad p=\sqrt{2\,m\,[E-U(x)]} \qquad \lambda=\frac{h}{\sqrt{2\,m\,[E-U(x)]}} \qquad \textit{Free Particle has } U(x)=0$$

$$\textit{Equation solution will be the Matter Particle Wavefunction } \Psi(x) \textit{ with Particle Probability Density } |\Psi^2(x)|$$

$$P(x_1\le x\le x_2)=\textit{Probability of particle between } x_1 \textit{ and } x_2=\int_{x_1}^{x_2}|\Psi^2(x)|\,dx \qquad \textit{where} \qquad \int_{-\infty}^{+\infty}|\Psi^2(x)|\,dx=1$$

### DeBroglie Wavelength and Matter Waves

The DeBroglie Matter Wavelength is the measurement for wave interactions such as interference of a mass $m$ particle. For large mass particles wavelengths are extremely small and effects are negligible, but are more dominant for particles.

$$\lambda_{matter\ wave}=\frac{h}{p_{object}}=\frac{h}{m_{object}\,v}=\frac{h}{\sqrt{2\,m_{object}\,K}} \qquad f_{matter\ wave}=\frac{p_{object}^2}{h\,m_{object}}=\frac{2\,K}{h} \quad \textit{where } K=\frac{1}{2}\,m_{object}\,v_{object}^2$$

### Heisenberg Uncertainty Principle

Uncertainty Principle restricts the uncertainty in simultaneous measurements of both position and momentum or both energy and time, and therefore position, momentum, and energy of a particle cannot all be known at any given instant.

$$\textit{Position and Momentum} \quad \Delta p_x\,\Delta x\gtrsim\hbar \qquad \Delta p_y\,\Delta y\gtrsim\hbar \qquad \Delta p_z\,\Delta z\gtrsim\hbar \qquad \textit{Energy and Time} \quad \Delta E\,\Delta t\gtrsim\hbar$$

$$\hbar=\frac{h}{2\pi}=1.055\times10^{-34}\,J\cdot s=6.582\times10^{-16}\,eV\cdot s$$

### One Dimensional Infinite Potential Well Trapped Particle

Particle trapped in a one dimensional width $L$ infinite potential well has a wave function with nodes at each wall.

$$\textit{Wave Function} \quad \Psi(x)=A\sin\left(\frac{n\,\pi\,x}{L}\right)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\,\pi\,x}{L}\right) \qquad \textit{Wavelength} \quad \lambda_{particle}=\frac{2L}{n}=\frac{h}{\sqrt{2\,m\,E}} \qquad n=1,2,3,4,\ldots$$

$$\textit{Probability Density} \quad \Psi^2(x)=\frac{2}{L}\sin^2\left(\frac{n\,\pi\,x}{L}\right) \qquad \textit{Quantized Energy Levels} \quad E_n=\left(\frac{h^2}{8\,m\,L^2}\right)n^2 \qquad n=1,2,3,4,\ldots$$

$$P(x_1\le x\le x_2)=\int_{x_1}^{x_2}\frac{2}{L}\sin^2\left(\frac{n\pi x}{L}\right)dx=\frac{2}{L}(x_2-x_1)-\frac{1}{n\pi}\left(\sin\left(\frac{2n\pi x_2}{L}\right)-\sin\left(\frac{2n\pi x_2}{L}\right)\right)\approx\frac{2}{L}\sin^2\left(\frac{n\pi x}{L}\right)(x_2-x_1)$$

$$\textit{Absorbed or Emitted Photon Energy, Wavelength, Frequency} \quad \Delta E=h\,f=\frac{h\,c}{\lambda_{photon}}=\left(\frac{h^2}{8\,m\,L^2}\right)(n_{high}^2-n_{low}^2)$$

$$m_{electron}=9.109\times10^{-31}\,kg \qquad h=6.63\times10^{-34}\,J\cdot s=4.14\times10^{-15}\,eV\cdot s \qquad c=3\times10^8\,\frac{m}{s}$$

### Multi Dimensional Infinite Potential Well Trapped Particle

Particle trapped in two dimensional widths $L_x$ $L_y$ infinite potential well has a wave function with nodes at each wall.

$$\textit{Quantized Energy Levels} \quad E_{n_x\,n_y}=\frac{h^2}{8\,m}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}\right) \qquad n_x=1,2,3,4,\ldots \qquad n_y=1,2,3,4,\ldots$$

$$\textit{Degenerate Levels occur when different sets of quantum numbers } n_x\ \ n_y \textit{ result in the same energy level}$$

Particle trapped in three dimensional widths $L_x$ $L_y$ $L_z$ infinite potential well has a wave function with nodes at each wall.

$$\textit{Quantized Energy Levels} \quad E_{n_x\,n_y\,n_z}=\frac{h^2}{8\,m}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right) \qquad n_x=1,2,3,4,\ldots \qquad n_y=1,2,3,4,\ldots \qquad n_z=1,2,3,4,\ldots$$

$$\textit{Degenerate Levels occur when different sets of quantum numbers } n_x\ \ n_y\ \ n_z \textit{ result in the same energy level}$$

## Finite Potential Well Trapped Particle, Finite Potential Barrier, and Barrier Tunneling

A particle trapped in a one dimensional width $L$ finite potential well or encountering a width $L$ finite potential barrier has a wave function with a exponentially decreasing small yet finite probability that the particle will transmit through by barrier tunneling to the other side. The probability of transmission or reflection depends on the barrier potential energy.

$$Transmission\ Coefficient \quad T = Average\ Probability\ of\ Particles\ that\ will\ Barrier\ Tunnel = e^{-2bL}$$

$$Reflection\ Coefficient \quad R = Average\ Probability\ of\ Particles\ that\ will\ Barrier\ Reflect = 1 - e^{-2bL}$$

$$b = \sqrt{\frac{8\,\pi^2\, m_{particle}\,(U_b - E)}{h^2}} \qquad U_b\ is\ the\ barrier\ potential\ energy\ and\ E\ is\ the\ particle\ energy$$

$$m_{electron} = 9.109 \times 10^{-31}\,kg \qquad m_{proton} = 1.673 \times 10^{-27}\,kg \qquad h = 6.63 \times 10^{-34}\,J \cdot s = 4.14 \times 10^{-15}\,eV \cdot s$$

## Spherical Potential Well Trapped Electron for the Hydrogen Atom

Particle trapped in a spherical potential well has a wave function that must overlap itself exactly per each circumference.

$$Potential\ Energy\ \ U(r) = -\frac{e^2}{4\,\pi\,\varepsilon_0\, r} = \frac{-2.307 \times 10^{-28}\,J \cdot m}{r} \qquad Angular\ Momentum\ \ L = n\,\hbar \qquad n = 1,2,3,4,\ldots$$

$$Bohr\ Radius\ of\ Hydrogen\ \ a = \frac{h^2\,\varepsilon_0\,n^2}{\pi\, m\, e^2} = \frac{h^2\,\varepsilon_0\,1^2}{\pi\, m\, e^2} = 52.92\,pm \qquad since\ n = 1\ for\ the\ ground\ state$$

$$Wave\ Function\quad \Psi(r) = \frac{1}{\sqrt{\pi\, a^3}}\, e^{-\left(\frac{r}{a}\right)} \qquad Probability\ Density\ \ \Psi^2(r) = \frac{4}{a^3}\, r^2\, e^{-\left(\frac{2r}{a}\right)}$$

$$P(r_1 \le r \le r_2) = \int_{r_1}^{r_2} \frac{4}{a^3}\, r^2\, e^{-\left(\frac{2r}{a}\right)}\, dr \qquad P(r_1 \le r \le r_2) \approx \frac{4}{a^3}\, r^2\, e^{-\left(\frac{2r}{a}\right)}(r_2 - r_1) \quad if\ \ r_1 \approx r_2$$

$$m_{electron} = 9.109 \times 10^{-31}\,kg \qquad h = 6.63 \times 10^{-34}\,J \cdot s = 4.14 \times 10^{-15}\,eV \cdot s \qquad a = 52.92 \times 10^{-12}\,m$$

Electron wave functions require it to exist only at certain quantized radii with defined probability density boundaries and have only certain quantized energy levels. An electron making a transition in energy level will emit or absorb a photon.

*The Principal Quantum Number n of an electron determines its quantized energy level and orbital radius*

$$Quantized\ Energy\ Levels\ \ E_n = -\frac{me^4}{8\,\varepsilon_0^2\,h^2}\,\frac{1}{n^2} = -\frac{2.180 \times 10^{-18}\,J}{n^2} = -\frac{13.60\,eV}{n^2} \qquad n = 1,2,3,4,\ldots$$

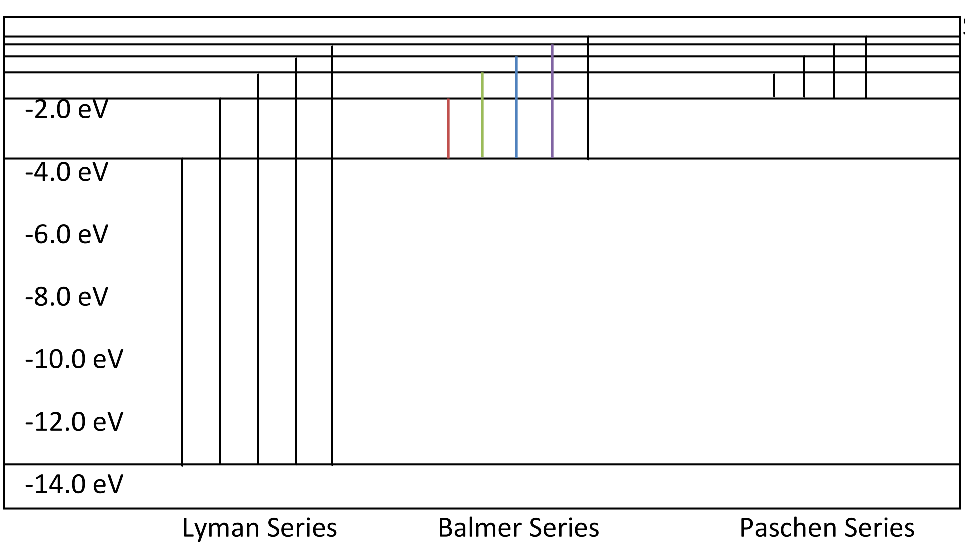
$$h = 6.63 \times 10^{-34}\,J \cdot s = 4.14 \times 10^{-15}\,eV \cdot s \qquad c = 3 \times 10^8\,\frac{m}{s}$$

$$\Delta E = h\,f = \frac{h\,c}{\lambda} = (2.180 \times 10^{-18}\,J)\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) = (13.60\,eV)\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) = (1.097 \times 10^7\,m^{-1})\left(\frac{1}{n_{low}^2} - \frac{1}{n_{high}^2}\right) \qquad since\ \ R = \frac{me^4}{8\,\varepsilon_0^2\,h^3 c} = 1.097 \times 10^7\,m^{-1}$$

Absorption of a photon by the atom occurs when the electron energy level is increased $\Delta E > 0$ as $n_f > n_i$

Emission of a photon by the atom occurs when the electron energy level is decreased $\Delta E < 0$ as $n_f < n_i$




Series are named by one value of $n$ as

$n = 3$ **Paschen Series**

Wavelengths are longer than visible

$n = 2$ **Balmer Series**

Wavelength Red for $n = 3$

Wavelength Green for $n = 4$

Wavelength Blue for $n = 5$

Wavelength Violet for $n = 6$

Wavelength Ultraviolet for $n > 6$

$n = 1$ **Lyman Series**

Wavelengths are shorter than visible

## Electrons and Electron Configuration

An atom consists of Nucleons (Protons and Neutrons) inside the nucleus surrounded by Electrons in stable orbitals.

$${}^{A}_{Z}X^{C}$$

$A$ is the number of Protons and Neutrons together, the nuclear mass in amu, the molar mass in g, or the atomic number
$Z$ is the number of Protons, or the Atomic Number that determines the element and the nuclear charge in terms of +e
$N = A - Z$ is the number of Neutrons in the nucleus
$E = Z - C$ is the number of Electrons in orbitals around the nucleus
$C = Z - E$ is the overall charge of the atom which may be either zero (Neutral), positive (Cation), or negative (Anion)
Ground State electrons will completely fill a lower energy orbital before filling the next highest energy orbital in order:

$$lowest \quad 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^6, 5s^2, 4d^{10}, 5p^6, 6s^2, 4f^{14}, 5d^{10}, 6p^6, 7s^2, 5f^{14}, 6d^{10}, 7p^6 \quad highest$$

| | $l$ | Names | Shapes | Number of Orbitals $(2l+1)$ | Number of Electrons $2(2l+1)$ |
|---|---|---|---|---|---|
| s | 0 | $s$ | 1 Sphere | 1 | 2 |
| p | 1 | $p_x, p_y, p_z$ | 3 Dumbbells | 3 | 6 |
| d | 2 | $d_{xy}, d_{xz}, d_{yz}, d_{x^2-y^2}, d_{z^2}$ | 4 Cloverleaves, 1 Dumbbell and Donut | 5 | 10 |
| f | 3 | | | 7 | 14 |

**f subshell** Each of the orbitals (boxes) can contain 2 spin paired electrons, one with spin up and one with spin down

$l = 3$

The $f$ subshell is filled through fourteen column Rare Earth elements
Rare Earth elements are found at the bottom of the Periodic Table

$m_l \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3$

**d subshell** Each of the orbitals (boxes) can contain 2 spin paired electrons, one with spin up and one with spin down

$l = 2$

The $d$ subshell is filled through ten column Transition Metal elements
Transition Metal elements are found in the middle of the Periodic Table

$m_l \quad -2 \quad -1 \quad 0 \quad +1 \quad +2$

**p subshell** Each of the orbitals (boxes) can contain 2 spin paired electrons, one with spin up and one with spin down

$l = 1$

The $p$ subshell is filled through six column Nonmetal elements
Nonmetal elements are found at the far right of the Periodic Table

$m_l \quad -1 \quad 0 \quad +1$

**s subshell** Each of the orbitals (boxes) can contain 2 spin paired electrons, one with spin up and one with spin down

$l = 0$

The $s$ subshell is filled through two column Alkali/Alkaline elements
Alkali/Alkaline elements are found at the far left of the Periodic Table

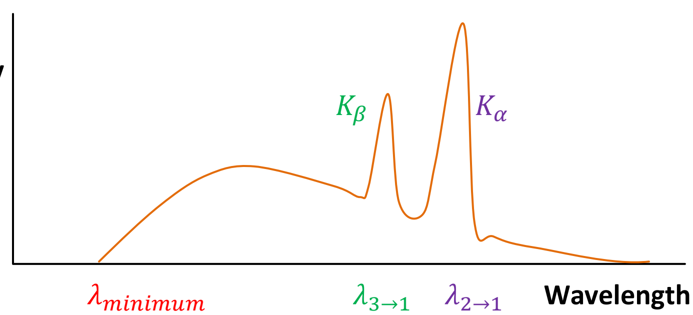
$m_l \quad 0$

X Ray Spectrum of Inner Core Electrons

High energy electron bombardment of an atom will will excite its inner core electrons. Other electrons quickly transition into the open spots, emitting X Rays.

$$\lambda_{minimum} = \frac{h\,c}{K_0} = \frac{h\,c}{q\,V_{potential}} \qquad K_0 = \frac{1}{2} m_{electron} v_{electron}^2$$

$K_\alpha$ *peak is* $2 \to 1$ *transition* $\quad K_\beta$ *peak is* $3 \to 1$ *transition*




Lasers

A Laser creates high intensity pulses of coherent light by first exciting electrons from a ground state to an excited state.

$N_0 = number\ of\ electrons\ in\ ground\ state\ energy\ E_0 \qquad N_x = number\ of\ electrons\ in\ excited\ state\ energy\ E_x$

$$\frac{N_x}{N_0} = e^{-\frac{(E_x - E_0)}{k\,T}} = e^{-\frac{(E_x - E_0)}{E_{average}}} \qquad \ln\left(\frac{N_x}{N_0}\right) = -\frac{(E_x - E_0)}{k\,T} = -\frac{(E_x - E_0)}{E_{average}} \qquad Population\ Inversion\ when\ N_x > N_0$$

$$Number\ of\ photons\ emitted\ in\ pulse\ \ N = N_x - N_0 \ \ when\ N_x > N_0 \qquad k = 1.381 \times 10^{-23}\frac{J}{K} = 8.617 \times 10^{-5}\frac{eV}{K}$$

$$Output\ Energy\ \ E_{output} = \frac{(N_x - N_0)\,h\,c}{\lambda} \qquad Power\ Output\ \ P_{output} = \frac{E_{output}}{t} = \frac{(N_x - N_0)\,h\,c}{\lambda\,t}$$

## Quantum Numbers

Each electron has a set of 4 quantum numbers $n$, $l$, $m_l$, and $m_s$, to describe its location and energy on an orbital diagram

| Symbol | Quantum Number Name | Allowed Values | Number of Possibilities |
|---|---|---|---|
| $n$ | Principle Quantum Number | $1,2,3,4,5,6,\ldots$ | |
| $l$ | Angular Momentum Quantum Number | $0,1,2,3,\ldots,n-1$ | $n$ |
| $m_l$ | Orbital Magnetic Quantum Number | $-l,\ldots,-2,-1,0,1,2,\ldots,l$ | $2l+1$ or $n^2$ |
| $m_s$ | Spin Magnetic Quantum Number | $-\frac{1}{2},\frac{1}{2}$ | 2 |

**The Pauli Exclusion Principle** states that only one electron can have a given combination of the four quantum numbers
For a ground state atom, the electrons will all exist in the lowest energy combination of the four quantum numbers
**Hunds Rule** states that for a given Angular Momentum Quantum Number $l$ a single electron will fill each orbital with the possible values of the Orbital Magnetic Quantum Number $m_l$ and identical signs for the Spin Magnetic Quantum Number $m_s$ before Spin Pairing a second electron with opposite sign of the Spin Magnetic Quantum Number $m_s$.
Atoms or Molecules with unpaired electrons in any orbitals will be Paramagnetic and be affected by a Magnetic Field
Atoms or Molecules with paired electrons in all orbitals will be Diamagnetic and not be affected by a Magnetic Field
**Angular Momentum and Magnetic Properties of an Electron** depend on its Momentum and Magnetic quantum number

$$Orbtial\ Angular\ Momentum \quad Magnitude\ L=\sqrt{l(l+1)}\,\hbar \quad l\ depends\ on\ subshell \quad Z\ component\ L_z=m_l\,\hbar$$

$$Orbital\ Magnetic\ Dipole\ Moment \quad \vec{\mu}_{orb}=-\frac{e}{2m}\vec{L} \quad \mu_{orb}=\frac{e}{2m}\sqrt{l(l+1)}\,\hbar \quad Z\ Component\ \mu_{orb\,z}=-m_l\,\mu_B$$

$$Spin\ Angular\ Momentum \quad Magnitude\ S=\sqrt{s(s+1)}\,\hbar=0.8660\,\hbar \quad since\ s=\frac{1}{2} \quad Z\ component\ S_z=m_s\,\hbar$$

$$Spin\ Magnetic\ Dipole\ Moment \quad \vec{\mu}_s=-\frac{e}{m}\vec{S} \quad \mu_s=\frac{e}{m}\sqrt{s(s+1)}\,\hbar \quad s=\frac{1}{2} \quad Z\ Component\ \mu_{s\,z}=-2m_s\,\mu_B$$

$$h\ bar\quad \hbar=\frac{h}{2\pi}=1.055\times10^{-34}\,J\cdot s=6.582\times10^{-16}\,eV\cdot s \qquad Bohr\ Magneton\quad \mu_B=\frac{e\,\hbar}{2\,m}=9.274\times10^{-24}\,\frac{J}{T}$$

## Unstable Nuclei and Radioactive Decay

**Alpha $\alpha$ Decay** heaviest isotopes (atomic number>83) beyond stability belt (excess nucleons or isotope weight>209)
A helium nucleus (alpha) 2 protons and 2 neutrons (both magic numbers) is emitted from the nucleus $2\,{}^1_1p+2\,{}^1_0n\rightarrow{}^4_2He$

$${}^A_ZX\rightarrow{}^{A-4}_{Z-2}L+{}^4_2He \qquad element\ L\ will\ be\ two\ units\ left\ of\ element\ X\ on\ periodic\ table$$

**Beta $\beta$ Decay** all isotopes above stability belt (excess neutrons or isotope weight>atomic weight)
A neutron is converted into a proton and an electron with the electron (beta) emitted from the nucleus ${}^1_0n\rightarrow{}^1_1p+{}^{\ 0}_{-1}e$

$${}^A_ZX\rightarrow{}_{Z+1}^{\ \ A}R+{}^{\ 0}_{-1}e \qquad element\ R\ will\ be\ one\ unit\ right\ of\ element\ X\ on\ periodic\ table$$

**Positron $e^+$ Decay** small isotopes (few electrons) below stability belt (excess protons or isotope weight<atomic weight)
A proton is converted into a neutron and a positron with the positron emitted from the nucleus ${}^1_1p\rightarrow{}^1_0n+{}^{\ 0}_{+1}e$

$${}^A_ZX\rightarrow{}_{Z-1}^{\ \ A}L+{}^{\ 0}_{+1}e \qquad element\ L\ will\ be\ one\ unit\ left\ of\ element\ X\ on\ periodic\ table$$

**Electron $e^-$ Capture** big isotopes (more electrons) below stability belt (excess protons or isotope weight<atomic weight)
A proton and an electron are converted into a neutron by a capture of an atoms own electron ${}^1_1p+{}^{\ 0}_{-1}e\rightarrow{}^1_0n$

$${}^A_ZX+{}^{\ 0}_{-1}e\rightarrow{}_{Z-1}^{\ \ A}L \qquad element\ L\ will\ be\ one\ unit\ left\ of\ element\ X\ on\ periodic\ table$$

**Gamma $\gamma$ Decay** occurs for isotopes rearranging nucleus orbitals without changing the number of protons or neutrons
The extra energy left after the rearrangement is carried away as a photon (gamma) from the nucleus ${}^A_ZX\rightarrow{}^A_ZX+{}^0_0\gamma$

$${}^A_ZX\rightarrow{}^A_ZX+{}^0_0\gamma \qquad element\ X\ will\ be\ unchanged\ but\ will\ have\ a\ more\ stable\ nucleus$$

**Decay Rate and Number of Nuclei**
Rate $rate_t$ for a large number of unstable nuclei decay is related to the final number $N_t$ or initial number $N_0$ of nuclei

$$rate_t=\frac{dN}{dt}=k\,N_t=k\,N_0\,e^{-k\,t} \qquad \frac{N_t}{N_0}=e^{-k\,t}=\left(\frac{1}{2}\right)^{\frac{t}{T_{half}}} \qquad \ln\left(\frac{N_t}{N_0}\right)=-kt=-\frac{0.693t}{T_{half}} \qquad T_{half}=\frac{\ln 2}{k}=\frac{0.693}{k}$$

**Binding Energy and Q Value**
The energy created within the binding of nucleons is directly related to the mass defect and the speed of light $c$

$$E_{binding}=\Delta m\,c^2=\left[m_{bonded\ object}-m_{unbonded\ components}\right]c^2 \qquad Q_{value}=\Delta m\,c^2=\left[m_{reactants}-m_{products}\right]c^2$$

$$m_{proton}=1.00783\,u \quad m_{neutron}=1.00867\,u \quad 1\,u=1.661\times10^{-27}\,kg \quad c=3.00\times10^8\,\frac{m}{s} \quad c^2=931.5\,\frac{MeV}{u}$$