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Equilibrium, Stress, Strain, and Strength of Materials

Equilibrium occurs when the total linear forces or stresses and angular forces or torques completely cancel to zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau_z = \sum r\, F \sin\theta = \sum (r\, F)_\perp = 0 \qquad \textit{Forces are determined from Free Body Diagram}$$

Axial Stress, Axial Strain, Thermal Strain, Relations and Compatibilities

Axial Stress is the perpendicular to surface total axial directed forces acting on an object divided by the cross sectional area of an object. Axial Stress creates Axial Strain or length change of an object divided by the initial length of the object. A temperature change creates Thermal Strain or length change of an object divided by the initial length of the object.

$$Axial\ Stress \quad \sigma = \frac{Total\ Axial\ Forces}{Cross\ Sectional\ Area} = \frac{F}{A} \qquad Axial\ Forces\ may\ be\ applied\ or\ due\ to\ temperature\ changes$$

$$Axial\ Strain \quad \epsilon = \frac{Change\ in\ Length}{Initial\ Length} = \frac{\Delta L_{axial}}{L_{initial}} = \frac{L_f - L_i}{L_i} \qquad Thermal\ Strain \quad \epsilon = \frac{\Delta L_{thermal}}{L_{initial}} = \frac{L_f - L_i}{L_i}$$

$$Total\ Strain\ and\ Percent\ Length\ Change \quad \%L\ change = \epsilon = \frac{\Delta L_{axial} + \Delta L_{thermal}}{L_{initial}} = \left(\alpha\left(T_{final} - T_{initial}\right) + \frac{F}{A\,E}\right)$$

Axial Stress and Axial Strain are directly proportional through material constant Youngs Modulus or Elastic Modulus E

$$\sigma = E\,\epsilon \qquad \frac{F}{A} = E\frac{\Delta L}{L} = E\left(\frac{L_{final} - L_{initial}}{L_{initial}}\right) \qquad \Delta L_{axial} = L_{final} - L_{initial} = \frac{F L_{initial}}{A\,E} \qquad L_{final} = L_{initial}\left(1 + \frac{F}{A\,E}\right)$$

Thermal Strain is directly proportional to temperature change through material constant α and may change axial forces

$$\Delta L_{thermal} = L_{final} - L_{initial} = \alpha\, L_{initial}\, \Delta T = \alpha\, L_{initial}\left(T_{final} - T_{initial}\right) \qquad \Delta T = T_{final} - T_{initial} = \frac{L_{final} - L_{initial}}{\alpha\, L_{initial}}$$

$$L_{final} = L_{initial}\left(1 + \alpha\left(T_{final} - T_{initial}\right)\right) \qquad T_{final} = T_{initial} + \frac{L_{final} - L_{initial}}{\alpha\, L_{initial}}$$

$$V_{final} = V_{initial}\left(1 + \beta\left(T_{final} - T_{initial}\right)\right) \qquad T_{final} = T_{initial} + \frac{V_{final} - V_{initial}}{\beta\, V_{initial}} \qquad \beta = 3\alpha$$

Combined applied axial forces and temperature changes are related to the initial length and final length of an object

$$L_{final} = L_{initial}\left(\frac{F}{A\,E} + \alpha\left(T_{final} - T_{initial}\right)\right) \qquad T_{final} = T_{initial} + \left(\frac{L_{final}}{L_{initial}} - \frac{F}{AE}\right)$$

Inline Compatibility relates length changes for multiple objects placed adjacent end on end with a possible gap between.

$$Inline\ No\ Gap\ Compatibility \quad (\Delta L_{thermal\,1} + \Delta L_{axial\,1}) + (\Delta L_{thermal\,2} + \Delta L_{axial\,2}) + (\Delta L_{thermal\,3} + \Delta L_{axial\,3}) = 0$$

$$Inline\ Gap\ Compatibility \quad (\Delta L_{thermal\,1} + \Delta L_{axial\,1}) + (\Delta L_{thermal\,2} + \Delta L_{axial\,2}) + (\Delta L_{thermal\,3} + \Delta L_{axial\,3}) = \delta_{gap}$$

Inset Compatibility relates length changes for two objects with one object placed enclosed inside the other object.

$$Inset\ No\ Gap\ Compatibility \qquad (\Delta L_{thermal\,1} + \Delta L_{axial\,1}) - (\Delta L_{thermal\,2} + \Delta L_{axial\,2}) = 0$$

Shear Stress, Shear Strain, and Relations

Shear Stress is the transverse to surface total shear directed forces acting on an object divided by the cross sectional area of an object. Shear Stress creates Shear Strain or angle change of an object vertex in radians where $180° = \pi\ rad$.

$$Shear\ Stress \quad \tau = \frac{Total\ Shear\ Forces}{Cross\ Sectional\ Area} = \frac{F}{A} \qquad Shear\ Strain \quad \gamma = Change\ in\ Radian\ Angle = \Delta\theta_{rad} = \theta_f - \theta_i$$

Shear Stress and Shear Strain are directly proportional through material constant Shear Modulus or Rigidity Modulus G

$$\tau = G\,\gamma \qquad \frac{F}{A} = G\,\Delta\theta_{rad} = G\left(\theta_{final} - \theta_{initial}\right) \qquad \Delta\theta = \theta_{final} - \theta_{initial} = \frac{F}{A\,G} \qquad \theta_{final} = \theta_{initial} + \frac{F}{A\,G}$$

Hydraulic Stress, Hydraulic Strain, and Relations

Hydraulic Stress is the Pressure or total radial directed forces acting on an object divided by the surface area of an object. Hydraulic Stress creates Axial Strain or volume change of an object divided by the initial volume of the object.

$$Hydraulic\ Stress \quad P = \frac{F}{A} = \frac{Total\ Radial\ Forces}{Surface\ Area} \qquad Hydraulic\ Strain \quad \nu = \frac{Change\ in\ Volume}{Initial\ Volume} = \frac{\Delta V}{V} = \frac{V_f - V_i}{V_i}$$

Hydraulic Stress Pressure and Hydraulic Strain are directly proportional through material constant Bulk Modulus B

$$P = B\,\nu \qquad P = \frac{F}{A} = B\frac{\Delta V}{V} = B\left(\frac{V_{final} - V_{initial}}{V_{initial}}\right) \qquad \Delta V = V_{final} - V_{initial} = \frac{P\, V_{initial}}{B} \qquad V_{final} = V_{initial}\left(1 + \frac{P}{B}\right)$$

Fluids

Fluids are liquids and gases which completely enclose an object with buoyancy pressure forces and flow through tubes.

$$P = \frac{Force\ on\ Fluid}{Boundary\ Area} = \frac{F}{A} \qquad P_{fluid\ open\ to\ atmosphere} = 1\ atm = 14.7\ psi = 101350\ Pa = 760\ mmHg = 760\ torr$$

Absolute Pressure and Gauge Pressure

$$Actual\ Pressure\ \ P_{absolute} = P_{atmosphere} + P_{gauge} \qquad Measured\ Gauge\ Pressure\ P_{gauge} = P_{absolute} - P_{atmosphere}$$

Pressure Due to Depth within a Fluid

$$P_{absolute} = P_{atmosphere} + P_{fluid\ at\ depth\ of\ h} = P_{atmosphere} + \rho_{fluid}\ g\ h_{depth\ of\ measurement} = P_{atm} + \rho\ g\ h$$

U Tube Manometer is a device used to measure pressures by the effect on liquid columns on both side of the u tube.

$$P_{absolute} = P_{atm} + \rho\ g\ h \quad Both\ Ends\ Open\ Tube\ \ \rho_1\ h_1 = \rho_2\ h_2 \quad One\ End\ Closed\ Tube\ \ P_1 + \rho_1\ g\ h_1 = \rho_2\ g\ h_2$$

Bouyancy

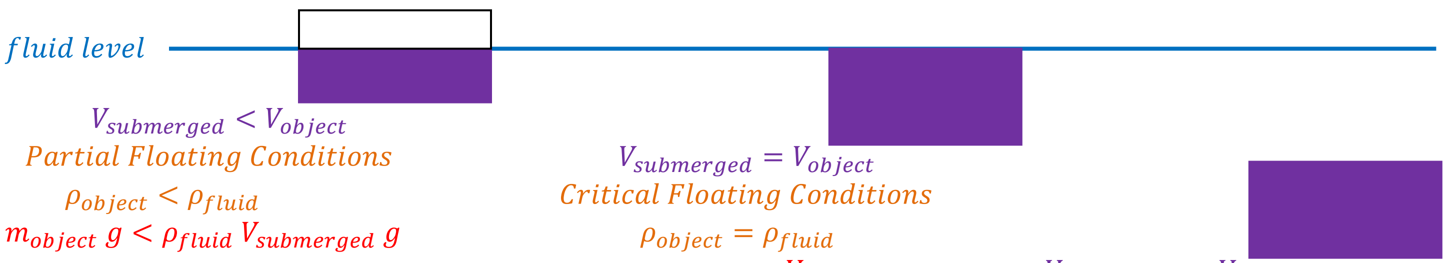
Bouyancy is the pressure force applied to an object residing partially or completely within a fluid of density ρ_{fluid}

$$Bouyancy\ Force\ \ F_{bouyancy} = \rho_{fluid}\ V_{submerged}\ g \qquad If\ object\ is\ completely\ submerged\ \ V_{submerged} = V_{object}$$

$$Weight\ Force\ \ W = m_{object}\ g = \rho_{object}\ V_{object}\ g \qquad where\ \ \rho_{object} = \frac{m_{object}}{V_{object}}$$

Partial Submerged Floating, Fully Submerged Critical Floating, and Sinking

Partial Submerged Floating occurs when upward buoyancy and other forces equal or exceed downward weight and other forces. Fully Submerged Critical Floating occurs when upward buoyancy and other forces equal downward weight and other forces. Sinking occurs when downward weight or other forces exceed upward buoyancy and other forces.



$V_{submerged} < V_{object}$
Partial Floating Conditions
$\rho_{object} < \rho_{fluid}$
$m_{object}\ g < \rho_{fluid}\ V_{submerged}\ g$

$V_{submerged} = V_{object}$
Critical Floating Conditions
$\rho_{object} = \rho_{fluid}$
$m_{object}\ g = \rho_{fluid}\ V_{submerged}\ g$

$V_{submerged} = V_{object}$
Sinking Conditions
$\rho_{object} > \rho_{fluid}$
$m_{object}\ g > \rho_{fluid}\ V_{submerged}\ g$

$$Percent\ Submerged\ \ \%Sub = \frac{V_{submerged}}{V_{object}} = 1\ if\ \rho_{object} \geq \rho_{fluid} \quad \%Sub = \frac{V_{submerged}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}}\ if\ \rho_{object} < \rho_{fluid}$$

$$Apparent\ Weight\ \ W_{app} = (\rho_{object}\ V_{object} - \rho_{fluid}\ V_{submerged})g\ \ if\ \rho_{object} > \rho_{fluid} \quad W_{app} = 0\ if\ \rho_{object} \leq \rho_{fluid}$$

Fluid Flow Equations

Fluid Flow Equations describe the relation between the speeds, heights, and pressures of a fluid at different locations.

Bernoulli Equation for Fluid Flow

$$P_1 + \frac{1}{2}\ \rho_1\ v_1^2 + \rho_1\ g\ y_1 = P_2 + \frac{1}{2}\ \rho_2\ v_2^2 + \rho_2\ g\ y_2$$

P_1 and P_2 is the pressure of the fluid at location 1 and 2
ρ_1 and ρ_2 is the density of the fluid at location 1 and 2
v_1 and v_2 is the speed of the fluid at location 1 and 2
y_1 and y_2 is the height of the fluid at location 1 and 2

Equation of Continuity for Fluid Flow

$$\rho_1\ A_1\ v_1 = \rho_2\ A_2\ v_2\ \ or\ \ \frac{dV_1}{dt} = A_2\ v_2\ \ or\ \ A_1\ v_1 = \frac{dV_2}{dt}$$

ρ_1 and ρ_2 is the density of the fluid at location 1 and 2
A_1 and A_2 is tube cross section area at location 1 and 2
v_1 and v_2 is the speed of the fluid at location 1 and 2
$\frac{dV_1}{dt}$ and $\frac{dV_2}{dt}$ is the volume flow rate at location 1 and 2

Force Transfer Formula is Bernoullis Equation for a contained static hydraulic force transfer from one end to another.

$$P_1 = P_2 \qquad which\ expands\ to \qquad \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad Where\ F_1\ is\ at\ one\ end\ and\ F_2\ is\ at\ another\ end$$

Toricellis Formula is Bernoullis Equation for fluid flow out of a large tank into the atmosphere through a small opening.

$$P_{above\ the\ fluid\ in\ the\ tank} + \rho_{fluid}\ g\ y_{height\ of\ fluid\ level\ above\ opening} = P_{atmosphere} + \frac{1}{2}\ \rho_{fluid}\ v_{out\ of\ the\ opening}^2$$

Wave Motion

Wave Motion is any kind of motion that repeats its exact transverse or longitudinal direction path over and over again at a fixed point of space in the same amount of time, this amount of time being known as the period T, while a wavefront oscillation between maximum and minimum values continues to move across the point in the longitudinal direction. Fluids, stretched strings, sound, and light are all examples of possible Wave Motion. Wave Motion has kinematic forms:

Elastic Displacement Wave Transverse Direction Relations

The Transverse Direction is the actual direction for the oscillation of mass particles as a wave front passes through.

Transverse Displacement

$y(x,t) = y_m \cos(k\,x \pm \omega\,t + \phi)$

$\pm$ will be positive for leftward moving waves

$\pm$ will be negative for rightward moving waves

Transverse Velocity

$v(x,t) = \frac{\partial y(x,t)}{\partial t} = \mp\,\omega\,y_m \sin(k\,x \pm \omega\,t + \phi)$

Transverse Acceleration

$a(x,t) = \frac{\partial v(x,t)}{\partial t} = -\omega^2\,y_m \cos(k\,x \pm \omega\,t + \phi)$

$a(x,t) = -\,\omega^2\,y(x,t)$

Transverse Force

$F(x,t) = -m\,\omega^2\,y_m \cos(k\,x \pm \omega\,t + \phi)$

$F(x,t) = -m\,\omega^2\,y(x,t)$

Transverse Displacement

$y(x,t) = y_m \sin(k\,x \pm \omega\,t + \phi)$

$\pm$ will be positive for leftward moving waves

$\pm$ will be negative for rightward moving waves

Transverse Velocity

$v(x,t) = \frac{\partial y(x,t)}{\partial t} = \pm\,\omega\,y_m \cos(k\,x \pm \omega\,t + \phi)$

Transverse Acceleration

$a(x,t) = \frac{\partial v(x,t)}{\partial t} = -\omega^2\,y_m \sin(k\,x \pm \omega\,t + \phi)$

$a(x,t) = -\,\omega^2\,y(x,t)$

Transverse Force

$F(x,t) = -m\,\omega^2\,y_m \sin(k\,x \pm \omega\,t + \phi)$

$F(x,t) = -m\,\omega^2\,y(x,t)$

y_m is displacement amplitude or the largest displacement x is fixed position along the wave front t is time elapsed

k is angular wave number ω is angular frequency or angular velocity ϕ is phase angle in radians $\tan\phi = -\frac{v(0,0)}{\omega\,y(0,0)}$

Maximum Transverse Displacement

$$y_m$$

Occurs with the following

$$displacement\ cos\ or\ sin = \pm 1$$
$$y(x,t) = max = +y_m\ \ or\ \ -y_m$$

or with the following

$$acceleration\ cos\ or\ sin = \pm 1$$
$$a(x,t) = max = +a_m\ \ or\ \ -a_m$$

or with the following

$$velocity\ sin\ or\ cos = 0$$
$$v(x,t) = 0$$

Maximum Transverse Velocity

$$v_m = \omega\,y_m$$

Occurs with the following

$$velocity\ cos\ or\ sin = \pm 1$$
$$v(x,t) = max = +v_m\ \ or\ \ -v_m$$

or with the following

$$displacement\ cos\ or\ sin = 0$$
$$y(x,t) = 0$$

or with the following

$$acceleration\ cos\ or\ sin = 0$$
$$a(x,t) = 0$$

Maximum Transverse Acceleration

$$a_m = \omega\,v_m = \omega^2\,y_m$$

Occurs with the following

$$acceleration\ cos\ or\ sin = \pm 1$$
$$a(x,t) = max = +a_m\ \ or\ \ -a_m$$

or with the following

$$displacement\ cos\ or\ sin = \pm 1$$
$$y(x,t) = max = +y_m\ \ or\ \ -y_m$$

or with the following

$$velocity\ sin\ or\ cos = 0$$
$$v(x,t) = 0$$

Longitudinal Direction Relations

The Longitudinal Direction is perpendicular to the Tranverse Direction and is the direction of wave propagation which has a wave speed v related to its angular frequency ω, wave number k,wavelength λ, period T, and cycle frequency f.

$$Longitudinal\ Wave\ Propagation\ Speed \qquad v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda\,f$$

Wave Motion Relations

Wave relations for wave speed v, angular frequency ω, wave number k,wavelength λ, period T, and cycle frequency f.

Natural Angular Frequency

$$\omega = 2\,\pi\,f = \frac{2\pi}{T}$$

Natural Cycle Frequency

$$f = \frac{\omega}{2\,\pi} = \frac{1}{T}$$

Natural Cycle Period

$$T = \frac{2\,\pi}{\omega} = \frac{1}{f}$$

Wave Number

$$k = \frac{2\,\pi}{\lambda}$$

Wavelength

$$\lambda = \frac{2\,\pi}{k}$$

$$\lambda = \frac{v}{f}$$

Sound Displacement Wave and Pressure Wave Longitudinal Direction Relations

The Longitudinal Direction is the direction for the motion of medium particles as a sound wave front passes through.

Longitudinal Pressure Difference

$\Delta p(x,t) = \Delta p_m \ \sin(k\,x \pm \omega\,t + \phi)$

$Amplitude\ Relations \quad \Delta p_m = v\,\rho\,\omega\,s_m$

$I = \frac{1}{2}\,\rho\,v\,\omega^2\,s_m^2 \qquad \rho\,\Delta p_m = B\,\rho_m$

$\pm$ will be positive for leftward moving waves

$\pm$ will be negative for rightward moving waves

Longitudinal Displacement

$s(x,t) = s_m \ \cos(k\,x \pm \omega\,t + \phi)$

$\pm$ will be positive for leftward moving waves

$\pm$ will be negative for rightward moving waves

Longitudinal Velocity

$v(x,t) = \frac{\partial s(x,t)}{\partial t} = \mp\,\omega\,s_m \ \sin(k\,x \pm \omega\,t + \phi)$

Longitudinal Acceleration

$a(x,t) = \frac{\partial v(x,t)}{\partial t} = -\omega^2\,s_m \cos(k\,x \pm \omega\,t + \phi)$

$a(x,t) = -\,\omega^2\,s(x,t)$

Longitudinal Force

$F(x,t) = -m\,\omega^2\,s_m \ \cos(k\,x \pm \omega\,t + \phi)$

$F(x,t) = -m\,\omega^2\,s(x,t)$

Longitudinal Pressure Difference

$\Delta p(x,t) = \Delta p_m \ \cos(k\,x \pm \omega\,t + \phi)$

$Amplitude\ Relations \quad \Delta p_m = v\,\rho\,\omega\,s_m$

$I = \frac{1}{2}\,\rho\,v\,\omega^2\,s_m^2 \qquad \rho\,\Delta p_m = B\,\rho_m$

$\pm$ will be positive for leftward moving waves

$\pm$ will be negative for rightward moving waves

Longitudinal Displacement

$s(x,t) = s_m \ \sin(k\,x \pm \omega\,t + \phi)$

$\pm$ will be positive for leftward moving waves

$\pm$ will be negative for rightward moving waves

Longitudinal Velocity

$v(x,t) = \frac{\partial y(x,t)}{\partial t} = \pm\,\omega\,s_m \ \cos(k\,x \pm \omega\,t + \phi)$

Longitudinal Acceleration

$a(x,t) = \frac{\partial v(x,t)}{\partial t} = -\omega^2\,s_m \sin(k\,x \pm \omega\,t + \phi)$

$a(x,t) = -\,\omega^2\,s(x,t)$

Longitudinal Force

$F(x,t) = -m\,\omega^2\,s_m \ \sin(k\,x \pm \omega\,t + \phi)$

$F(x,t) = -m\,\omega^2\,s(x,t)$

s_m is displacement amplitude $\quad \Delta p_m$ is pressure amplitude $\quad x$ is fixed position along the wave front $\quad t$ is time elapsed

k is angular wave number $\quad \omega$ is angular frequency or angular velocity $\quad \phi$ is phase angle in radians $\quad \tan\phi = -\frac{v(0,0)}{\omega\,y(0,0)}$

Maximum Sound Pressure

Δp_m

Occurs with the following

$pressure\ cos\ or\ sin = \pm 1$

$p(x,t) = max = +\Delta p_m \ \ or\ \ -\Delta p_m$

$displacement\ cos\ or\ sin = 0$

$y(x,t) = 0$

or with the following

$acceleration\ cos\ or\ sin = 0$

$a(x,t) = 0$

or with the following

$velocity\ cos\ or\ sin = \pm 1$

$v(x,t) = max = +v_m \ \ or\ \ -v_m$

Maximum Transverse Velocity

$v_m = \omega\,y_m$

Occurs with the following

$velocity\ cos\ or\ sin = \pm 1$

$v(x,t) = max = +v_m \ \ or\ \ -v_m$

or with the following

$pressure\ cos\ or\ sin = \pm 1$

$p(x,t) = max = +\Delta p_m \ \ or\ \ -\Delta p_m$

$displacement\ cos\ or\ sin = 0$

$y(x,t) = 0$

or with the following

$acceleration\ cos\ or\ sin = 0$

$a(x,t) = 0$

Maximum Transverse Acceleration

$a_m = \omega\,v_m = \omega^2\,y_m$

Occurs with the following

$acceleration\ cos\ or\ sin = \pm 1$

$a(x,t) = max = +a_m \ \ or\ \ -a_m$

or with the following

$displacement\ cos\ or\ sin = \pm 1$

$y(x,t) = max = +y_m \ \ or\ \ -y_m$

$pressure\ \cos\ or\ \sin = 0$

$p(x,t) = 0$

or with the following

$velocity\ sin\ or\ cos = 0$

$v(x,t) = 0$

Longitudinal Direction Relations

Wave speed v is related to its angular frequency ω, wave number k,wavelength λ, period T, and cycle frequency f.

$$Longitudinal\ Wave\ Propagation\ Speed \quad v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda\,f \qquad Arrival\ Time\ Difference \quad \Delta t = d\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$$

Wave Motion Relations

Wave relations for wave speed v, angular frequency ω, wave number k,wavelength λ, period T, and cycle frequency f.

Natural Angular Frequency

$$\omega = 2\,\pi\,f = \frac{2\pi}{T}$$

Natural Cycle Frequency

$$f = \frac{\omega}{2\,\pi} = \frac{1}{T}$$

Natural Cycle Period

$$T = \frac{2\,\pi}{\omega} = \frac{1}{f}$$

Wave Number

$$k = \frac{2\,\pi}{\lambda}$$

Wavelength

$$\lambda = \frac{2\,\pi}{k}$$

$$\lambda = \frac{v}{f}$$

Phase Difference, Location Difference, Time Difference on a Single Wave

Phase Difference $\Delta\phi$ is the angular difference in radians between two points that differ in location or in time on a wave.

Location Difference

$\Delta x = \frac{\Delta\phi}{2\pi}\,\lambda$

Δx Location difference between two points on a wave

λ Wavelength of the wave

Time Difference

$\Delta t = \frac{\Delta\phi}{2\pi}\,T$

Δt Time difference between two points on a wave

T Period of the wave

Interference of Two Waves

Interference of Wave motion results when two wave fronts with matching amplitude, angular frequency, and angular wave number but phase shifted interfere with each other and combine to form a new wave front form

$$y(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t + \phi) = 2\,y_m \cos\left(\frac{1}{2}\phi\right)\,\sin\left(kx + \omega t + \frac{1}{2}\phi\right) \qquad y_{max} = 2\,y_m \cos\left(\frac{1}{2}\phi\right)$$

$$Arrival\ Time\ Difference\ of\ Wave\ Speeds \quad \Delta t = d\left(\frac{1}{v_2} - \frac{1}{v_1}\right) \qquad Speed\ of\ Sound \quad v = (331 + 0.6\,T_{in\,^\circ C})\frac{m}{s}$$

Constructive Interference of Waves Antinode Maxima

$Path\ difference = x_2 - x_1 = m\,\lambda = \frac{mv}{f} \quad m = 1,2,\ldots$

Destructive Interference of Waves Node Minima

$Path\ difference = x_2 - x_1 = \frac{(2m-1)\lambda}{2} = \frac{(2m-1)v}{2f} \quad m = 1,2,\ldots$

Standing Waves

Standing Wave motion results when two wave fronts with matching amplitude, angular frequency, and angular wave number but moving in opposite directions interfere with each other and combine to form a new wave front form

$$y(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) = 2\,y_m\,\sin(kx)\cos(\omega t) \qquad y_{max} = 2\,y_m \sin(kx)$$

$$v(x,t) = \frac{\partial y(x,t)}{\partial t} = -2\,\omega\,y_m\,\sin(kx)\sin(\omega t) \qquad a(x,t) = \frac{\partial v(x,t)}{\partial t} = -2\,\omega^2\,y_m\,\sin(kx)\cos(\omega t)$$

The location of the nodes (zeros) and the antinodes (maxima) are stationary and occur only at certain allowed values

Allowed Node (Zero) Location, Wavelength, Frequency

$x = \frac{n\,\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = \frac{n\,v}{2\,L}$

n is any positive integer: $n = 1,2,3,4,\ldots$

Allowed Antinode (Max) Location, Wavelength, Frequency

$x = \frac{(2n-1)\,\lambda}{4} \qquad \lambda = \frac{4L}{(2n-1)} \qquad f = \frac{(2n-1)v}{4\,L}$

The maximum amplitude achieved by the antinode locations is $2y_m$

Stretched String Sound Waves

Waves that travel on a stretched string with both ends clamped have a speed related to the Tension T in the string

$$Speed\ of\ String\ Wave \quad v = \sqrt{\frac{T}{\mu}} \qquad Power\ of\ String\ Wave \quad P_{avg} = \frac{1}{2}\,\mu\,v\,\omega^2\,y_m^2 \qquad with\ Linear\ Density\ \mu = \frac{m}{L}$$

T is string Tension, which is known or found from Newtons Second Law

μ is the Linear Mass Density $\mu = \frac{m}{L}$

Waves on a stretched string must have both ends as nodes and only certain wavelengths and frequencies are allowed.

Allowed Wavelengths

$\lambda = \frac{2L}{n}$

n is any positive integer: $n = 1,2,3,4,\ldots$

Allowed Frequencies

$f = \frac{v}{\lambda} = \frac{nv}{2L}$

$n = 1$ is the fundamental frequency and the fundamental wavelength

Open or Closed End Pipe Sound Waves

Waves that travel in a pipe with open or closed ends have a speed equal to the speed of sound in air at temperature T

$$Speed\ of\ Pipe\ Wave\ at\ 20°C \quad v = 343\frac{m}{s} \qquad Speed\ of\ Pipe\ Wave\ at\ any\ Temp\ T \quad v = (331 + 0.6\,T_{in\,^\circ C})\frac{m}{s}$$

Waves must have closed end nodes and open end antinodes and only certain wavelengths and frequencies are allowed.

Pipe with Both Ends Open or Pipe with Both Ends Closed
Allowed Displacement Node (Zero) Location, Pressure Antinode (Maximum) Location, Wavelength, Frequency

$x = \frac{n\,\lambda}{2} \qquad \lambda = \frac{2L}{n} \qquad f = \frac{n\,v}{2\,L}$

n is any positive integer: $n = 1,2,3,4,\ldots$

Pipe with One End Open and One End Closed
Allowed Displacement Antinode (Maximum) Location, Pressure Node (Zero) Location, Wavelength, Frequency

$x = \frac{(2n-1)\,\lambda}{4} \qquad \lambda = \frac{4L}{(2n-1)} \qquad f = \frac{(2n-1)v}{4\,L}$

$n = 1$ is the fundamental frequency and the fundamental wavelength

Beat Interference of Two Waves

Beat Interference of Wave motion results when two wave fronts with matching amplitude, angular wave number, and zero phase shift but with different angular frequencies interfere with each other and combine to form a new wave front

$$y(x,t) = y_m \sin(kx - \omega_1 t) + y_m \sin(kx + \omega_2 t) = 2\, y_m\ \sin(kx)\ \cos\left(\left(\frac{\omega_1 - \omega_2}{2}\right) t\right)\ \cos\left(\left(\frac{\omega_1 + \omega_2}{2}\right) t\right)$$

$$y(x,t) = a_m \cos\left(\left(\frac{\omega_1 + \omega_2}{2}\right) t\right) \qquad \textit{with amplitude} \qquad a_m = 2\, y_m\ \sin(kx)\ \cos\left(\left(\frac{\omega_1 - \omega_2}{2}\right) t\right)$$

The resulting wave front has an amplitude that varies in time with a Beat Cycle Frequency and Beat Angular Frequency

$$\textit{Amplitude Beat Cycle Frequency} \quad f_{beat} = f_1 - f_2 \qquad \textit{Amplitude Beat Angular Frequency} \quad \omega_{beat} = \omega_1 - \omega_2$$

Doppler Effect

Doppler Effect occurs when a wave with speed $v_{wave\ in\ medium}$ is either emitted from a moving source v_{source} received by a moving detector $v_{detector}$ or both. The received wave cycle frequency $f_{detector}$ and wavelength $\lambda_{detector}$ at the detector is related to the emitted wave cycle frequency f_{source} and wavelength λ_{source} from the source

$$f_{detector} = f_{source} \left(\frac{v_{wave\ in\ medium} \pm v_{detector}}{v_{wave\ in\ medium} \mp v_{source}}\right) \qquad \textit{for a different source and detector}$$

$$\lambda_{source} = \lambda_{detector} \left(\frac{v_{wave\ in\ medium} \pm v_{detector}}{v_{wave\ in\ medium} \mp v_{source}}\right) \qquad \textit{for a different source and detector}$$

$\textit{Numerator sign for } v_{detector}$ $\quad + \textit{for detector approaching source} \quad - \textit{for detector receding from source}$

$\textit{Denominator sign for } v_{source}$ $\quad - \textit{for source approaching detector} \quad + \textit{for source receding from detector}$

Detecting echo wave $v_{wave\ in\ medium}$ from object v_{object} the received wave cycle frequency $f_{detector}$ and wavelength $\lambda_{detector}$ at the detector is related to the emitted wave cycle frequency f_{source} and wavelength λ_{source} from the source

$$f_{detector} = f_{source} \left(\frac{v_{wave\ in\ medium} \pm v_{object}}{v_{wave\ in\ medium} \mp v_{source}}\right) \left(\frac{v_{wave\ in\ medium} \pm v_{source}}{v_{wave\ in\ medium} \mp v_{object}}\right) \qquad \textit{source detector echo from object}$$

$$\lambda_{source} = \lambda_{detector} \left(\frac{v_{wave\ in\ medium} \pm v_{object}}{v_{wave\ in\ medium} \mp v_{source}}\right) \left(\frac{v_{wave\ in\ medium} \pm v_{source}}{v_{wave\ in\ medium} \mp v_{object}}\right) \qquad \textit{source detector echo from object}$$

$\textit{Numerator sign for } v_{object}$ $\quad + \textit{for object approaching source} \quad - \textit{for object receding from source}$

$\textit{Denominator sign for } v_{object}$ $\quad - \textit{for object approaching source} \quad + \textit{for object receding from source}$

$\textit{Denominator sign for } v_{source}$ $\quad - \textit{for source approaching object} \quad + \textit{for source receding from object}$

$\textit{Numerator sign for } v_{source}$ $\quad + \textit{for source approaching object} \quad - \textit{for source receding from object}$

Sound Source Power, Sound Intensity, Sound Detector Power, and Decibel Level

Sound Source Power is the rate at which energy is emitted from the sound source per unit time. The sound emanates outward usually in either a spherical shell wave front or a hemispherical shell wave front and has a received intensity

$$\textit{Sound Intensity} \qquad I_{received} = \frac{P_{source}}{A_{wavefront\ area}} \qquad \textit{where } A_{wavefront\ area} \textit{ may depend on distance}$$

$$A = 4\pi r^2 \ \ \textit{isotropic spherical wavefront at distance } r \qquad A = 2\pi r^2 \ \ \textit{hemispherical wavefront at distance } r$$

$$\textit{Sound Detector Power} \qquad P_{received} = \frac{P_{source}\, A_{receiver}}{A_{wavefront\ area}} \qquad \textit{where } A_{receiver} \textit{ is the receiver cross sectional area}$$

Since Sound Intensity exists on a very large range of values it is common to reduce the scale size on the log Decibel Level

$$\textit{Decibel Level} \qquad d\beta = 10 \log\left(\frac{I}{I_0}\right) \qquad \textit{where the threshold of hearing standard intensity is} \quad I_0 = 1 \times 10^{-12}\, \frac{W}{m^2}$$

$$\textit{Difference in Decibel Level for two sources with intensities } I_1 \textit{ and } I_2 \qquad \Delta\beta = \beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$$

Mach Number and Mach Cone Angle

Mach Number N is the ratio of an object speed to the speed of sound in the medium within which the object moves. Mach Cone Angle θ_m is the angle of the shockwave created by an object exceeding the speed of sound in the medium.

$$\textit{Mach Number} \qquad N = \frac{v_{object\ in\ medium}}{v_{soundin\ medium}} \qquad \textit{Mach Cone Angle} \qquad \sin\theta_m = \frac{1}{N} = \frac{v_{sound\ in\ medium}}{v_{objectin\ medium}}$$

Kinetic Theory of Gases

Kinetic Theory of gas describes the relations of the state variables pressure, volume, and temperature to the gas speeds.

Ideal Gas Laws

Many gases approximate the Ideal Gas state. A gas must meet two conditions to be ideal and follow the Ideal Gas Laws:

$$\textit{Gas molecules must occupy a very small fraction of the container volume by existing at a low pressure.}$$

$$\textit{Gas molecules must have high energy and small intermolecular forces by existing at a high temperature.}$$

Ideal Gas Laws relate state variables pressure P, volume V, and temperature T to amount of gas in moles or molecules.

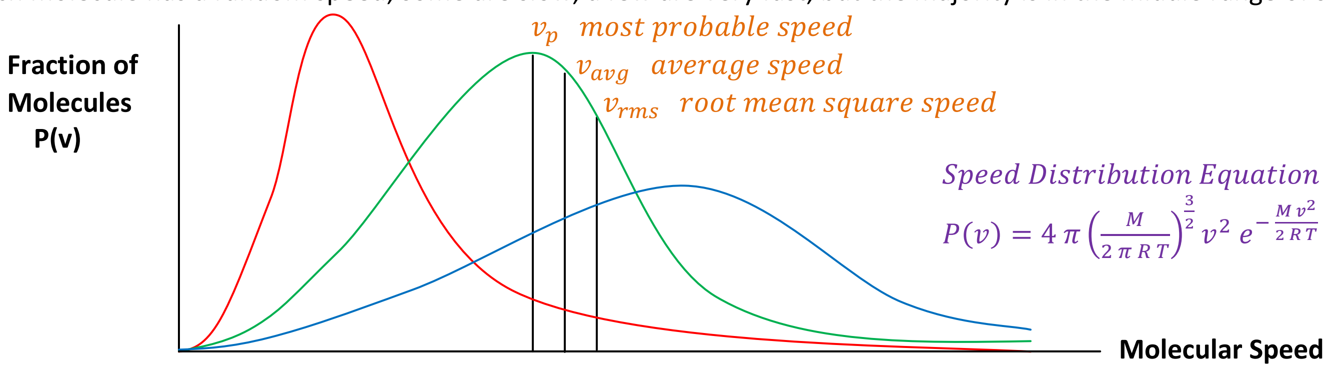
$$Moles\ n\ Version\quad P\,V = n\,R\,T \qquad Molecules\ N\ Version\quad P\,V = N\,k\,T \qquad State\ Variable\ Relation\quad \frac{P_1\,V_1}{n_1\,T_1} = \frac{P_2\,V_2}{n_2\,T_2}$$

$$Pressure\ 1\ atm = 14.7\ psi = 101350\ Pa \quad Temperature\ K = ℃ + 273 \quad ℃ = \frac{5}{9}(℉ - 32) \quad Volume\ 1\ m^3 = 1000\ L$$

$$n = \frac{m}{M}\ \frac{mass}{molar\ mass} \qquad Molecules\ N = n\,N_a\ where\ N_a = 6.022\times 10^{23} \qquad R = 8.314\frac{J}{mol\ K} \qquad k = 1.381\times 10^{-23}\frac{J}{K}$$

Molecular Speed Distribution and Root Mean Square Speed

Each molecule has a random speed; some are slow, a few are very fast, but the majority is in the middle range of speeds



$$Speed\ Distribution\ Equation$$

$$P(v) = 4\,\pi\left(\frac{M}{2\,\pi\,R\,T}\right)^{\frac{3}{2}} v^2\,e^{-\frac{M\,v^2}{2\,R\,T}}$$

The above colored coded graphs can be compared to each other according to the following schemes:

Absolute Temperature

The higher absolute temperature in Kelvin, the faster individual molecules move and the more the curve is stretched

$$Temperature\ of\ Blue\ Curve > Temperature\ of\ Green\ Curve > Temperature\ of\ Red\ Curve$$

Molecular Weight

The higher molecular weight in grams, the slower individual molecules move and the more the curve is compressed

$$Molecular\ Weight\ of\ Red\ Curve > Molecular\ Weight\ of\ Green\ Curve > Molecular\ Weight\ of\ Blue\ Curve$$

Most Probable Speed, Average Speed, Root Mean Square Speed

Most Probable Speed v_p is the maximum of the distribution curve, Average Speed v_{avg} is the statistical mean of the distribution curve, and the Root Mean Square Speed v_{rms} is the root of the mean of the squared distribution curve.

$$Root\ Mean\ Square\ Speed\ v_{rms} = \sqrt{\frac{3\,R\,T}{M}} \qquad Average\ Speed\ v_{avg} = \sqrt{\frac{8\,R\,T}{\pi\,M}} \qquad Most\ Probable\ Speed\ v_p = \sqrt{\frac{2\,R\,T}{M}}$$

$$Normalization\ \int_0^{\infty} P(v)\,dv = 1 \qquad Average\ Speed\ v_{avg} = \int_0^{\infty} v\,P(v)\,dv \qquad Most\ Probable\ Speed\ P'(v_p) = 0$$

Kinetic Energy per Molecule and Kinetic Energy Total

Each gas molecule has random speed and kinetic energy, but average kinetic energy that depends only on temperature.

$$K_{average\ per\ molecule} = \frac{3\,R\,T}{2\,N_A} = \frac{3}{2}\,k\,T \qquad K_{total} = N\,K_{average\ per\ molecule} = \frac{3}{2}\,N\,k\,T = \frac{1}{2}\,m\,v_{rms}^2 = \frac{3}{2}\,n\,R\,T$$

Mean Free Path

Mean Free Path is the average distance a gas molecule travels between collisions and is inversely related to density.

$$\lambda_{mean\ free\ path} = \frac{1}{\sqrt{2}\,\pi\,d^2\,\frac{N}{V}} = \frac{k\,T}{\sqrt{2}\,\pi\,d^2\,P} \qquad d = molecule\ diameter \qquad \frac{N}{V} = number\ of\ molecules\ per\ volume$$

Laws of Thermodynamics

Thermodynamics is the study of heat transfer, work of a system, stored internal energy, and order energy or entropy.

Zeroth Law The steady state equilibrium of objects in contact require the temperatures to be equal $T_{object\ 1} = T_{object\ 2}$

First Law The relationship between the change in internal energy ΔE, heat Q added to, and work W done by a system

$$State\ Relation \quad \Delta E = Q + W \qquad Differential\ Relation \quad dE = dQ + dW$$

Second Law The total entropy or disorder of the Universe will always increase during any process

$$\Delta S_{universe} > 0 \qquad where \qquad \Delta S_{universe} = \Delta S_{system} + \Delta S_{surroundings}$$

Third Law The entropy of a substance will only be zero when the substance is in a pure crystalline form at absolute zero

$$S^{\circ} = 0 \quad only\ when\ in\ a\ pure\ crystalline\ form\ and\ at\ absolute\ zero\ temperature\ T = 0\ K$$

Internal Energy, Heat, Work, Entropy and Gases in the First Law of Thermodynamics

First Law is the Work Energy Equation change in internal energy ΔE, heat Q added to, and work W done by the system.

$$State\ Relation \quad \Delta E = Q + W \qquad Differential\ Relation \quad dE = dQ + dW$$

Internal Energy Change To System

$$dE = dQ + dW = n\,C_v\,dT$$
$$\Delta E = n\,C_v\,\Delta T$$

Heat Added To The System

$$dQ = dE - dW = n\,C_v\,dT + p\,dV$$
$$Q = \Delta E - W = n\,C_v\,\Delta T + \int p\,dV$$

Work Done On The System

$$dW = dE - dQ = -p\,dV$$
$$W = -\int p\,dV = -Area\ below\ PV\ curve$$

$$Isothermal\ (\Delta T = 0) \quad \Delta E = nC_v\Delta T = 0 \quad W = -n\,R\,T\,\ln\left(\frac{V_{final}}{V_{initial}}\right) \quad Q = -W = n\,R\,T\,\ln\left(\frac{V_{final}}{V_{initial}}\right)$$

$$Isobaric\ (\Delta P = 0) \quad \Delta E = nC_v\Delta T \quad W = -p\Delta V \quad Q = nC_v\Delta T + p\Delta V = nC_v\Delta T + nR\Delta T = nC_p\Delta T$$

$$Isochoric\ (\Delta V = 0) \quad \Delta E = nC_v\Delta T \quad W = 0 \quad Q = \Delta E = nC_v\Delta T$$

$$Adiabatic\ (Q = 0) \quad \Delta E = nC_v\Delta T \quad W = \Delta E = nC_v\Delta T \quad Q = 0$$

$$Free\ Expansion\ (P = 0\ and\ Q = 0\ but\ \Delta V > 0) \quad \Delta E = nC_v\Delta T = 0 \quad W = 0 \quad Q = 0$$

$$Full\ Cycle\ (\Delta T = 0\ and\ \Delta P = 0\ and\ \Delta V = 0) \quad \Delta E = nC_v\Delta T = 0$$

$$C_v = \left(\frac{degrees\ of\ freedom\ f}{2}\right)R \qquad C_p = C_v + R$$

monatomic gas

$$C_v = \frac{3}{2}R \qquad C_p = C_v + R = \frac{5}{2}R$$

diatomic gas

$$C_v = \frac{5}{2}R \qquad C_p = C_v + R = \frac{7}{2}R$$

polyatomic gas

$$C_v = 3R \qquad C_p = C_v + R = 4R$$

Entropy

Entropy is the state function of disorder energy per temperature as a natural log of all possible microstate combinations

$$Entropy\ \ S = k\ln W \qquad k = 1.381 \times 10^{-23}\frac{J}{K} \qquad W = \frac{N!}{n_1!\,n_2!\,n_3!\,\ldots} \quad with\ total\ microstates\ N = n_1 + n_2 + n_3 + \cdots$$

$$Stirlings\ Approximation \quad \ln N! = N\ln N - N \quad and \quad S = k[(N\ln N - N) - (n_1\ln n_1 - n_1) - (n_2\ln n_2 - n_2) - \cdots]$$

Entropy Change ΔS_{irr} for an irreversible process is equal to Entropy Change ΔS_{rev} of any equivalent reversible process

$$\Delta S = \Delta S_{irreversible} = \Delta S_{reversible} = \int_{initial\ state}^{final\ state}\frac{dQ}{T} \qquad dQ = T\,dS \qquad Q = \int_{initial\ state}^{final\ state} T\,dS$$

$$\Delta S = 0 \quad Adiabatic \qquad \Delta S = \frac{Q}{T} \quad Isothermal \qquad \Delta S = \frac{Q}{T} \quad Heat\ Resevoir \qquad \Delta S = k\,\ln\left(\frac{W_{final}}{W_{initial}}\right) \quad Any\ Process$$

$$\Delta S = \frac{m\,L}{T} \quad Solid, Liquid, or\ Gas\ Phase\ Change \qquad \Delta S = m\,c\,\ln\left(\frac{T_{final}}{T_{initial}}\right) \quad Solid\ or\ Liquid\ Temperature\ Change$$

$$\Delta S = n\,R\,\ln\left(\frac{V_{final}}{V_{initial}}\right) + n\,C_V\,\ln\left(\frac{T_{final}}{T_{initial}}\right) = n\,R\ln\left(\frac{V_{final}}{V_{initial}}\right) + n\,C_V\,\ln\left(\frac{P_{final}\,V_{final}}{P_{initial}\,V_{initial}}\right) \quad Gas\ for\ Any\ Process$$

$$C_v = \frac{3}{2}R \quad monatomic\ gas \qquad C_v = \frac{5}{2}R \quad diatomic\ gas \qquad C_v = 3R \quad polyatomic\ gas$$

Processes with PV Diagrams and Equations for Internal Energy, Heat, Work, and Entropy

Isothermal $T = constant \quad p_{final}\,V_{final} = p_{initial}\,V_{initial}$

$Internal\ Energy \quad \Delta E = 0$

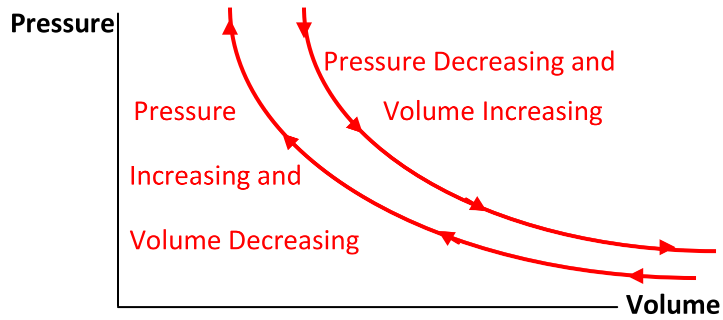
$Heat \quad Q = n\,R\,T\,\ln\left(\frac{V_{final}}{V_{initial}}\right) = n\,R\,T\,\ln\left(\frac{p_{initial}}{p_{final}}\right)$

$Q = p\,V\,\ln\left(\frac{V_{final}}{V_{initial}}\right) = p\,V\,\ln\left(\frac{p_{initial}}{p_{final}}\right)$

$Work \quad W = -n\,R\,T\,\ln\left(\frac{V_{final}}{V_{initial}}\right) = -n\,R\,T\,\ln\left(\frac{p_{initial}}{p_{final}}\right)$

$W = -p\,V\,\ln\left(\frac{V_{final}}{V_{initial}}\right) = -p\,V\,\ln\left(\frac{p_{initial}}{p_{final}}\right)$

$Entropy \quad \Delta S = \frac{Q}{T}$



Isochoric $V = constant \qquad \frac{p_{final}}{T_{final}} = \frac{p_{initial}}{T_{final}}$

$Internal\ Energy \quad \Delta E = n\,C_v\,(T_{final} - T_{initial})$

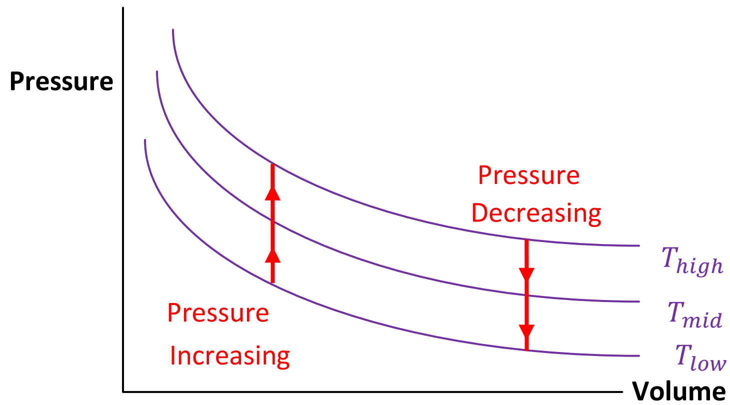
$Heat \quad Q = n\,C_v\,(T_{final} - T_{initial})$

$Work \quad W = 0$

$Entropy \quad \Delta S = n\,C_V\,\ln\left(\frac{T_{final}}{T_{initial}}\right) = n\,C_V\,\ln\left(\frac{p_{final}}{p_{initial}}\right)$

$monatomic\ gas \quad C_v = \frac{3}{2}R \qquad diatomic\ gas \quad C_v = \frac{5}{2}R$

$polyatomic\ gas \quad C_v = 3R$



Isobaric $p = constant \qquad \frac{V_{final}}{T_{final}} = \frac{V_{initial}}{T_{final}}$

$Internal\ Energy \quad \Delta E = n\,C_v\,(T_{final} - T_{initial})$

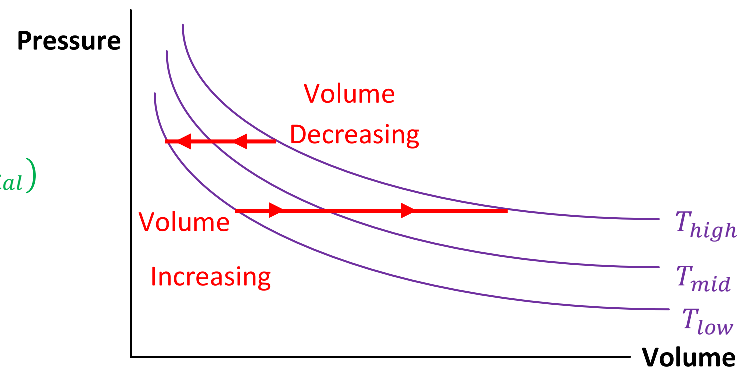
$Heat \quad Q = n\,C_p\,(T_{final} - T_{initial})$

$Work \quad W = -p(V_{final} - V_{initial}) = -n\,R(T_{final} - T_{initial})$

$Entropy \quad \Delta S = n\,R\,\ln\left(\frac{V_{final}}{V_{initial}}\right) + n\,C_v\,\ln\left(\frac{T_{final}}{T_{initial}}\right)$

$monatomic\ gas \quad C_v = \frac{3}{2}R \qquad diatomic\ gas \quad C_v = \frac{5}{2}R$

$polyatomic\ gas \quad C_v = 3R \qquad C_p = C_v + R$



Adiabatic $q = 0 \quad p_{final}\,V_{final}^{\gamma} = p_{initial}\,V_{initial}^{\gamma}$

$T_{final}\,V_{final}^{\gamma-1} = T_{initial}\,V_{initial}^{\gamma-1} \qquad \gamma = \frac{C_p}{C_v} = \frac{C_v+R}{C_v}$

$Internal\ Energy \quad \Delta E = n\,C_v\,(T_{final} - T_{initial})$

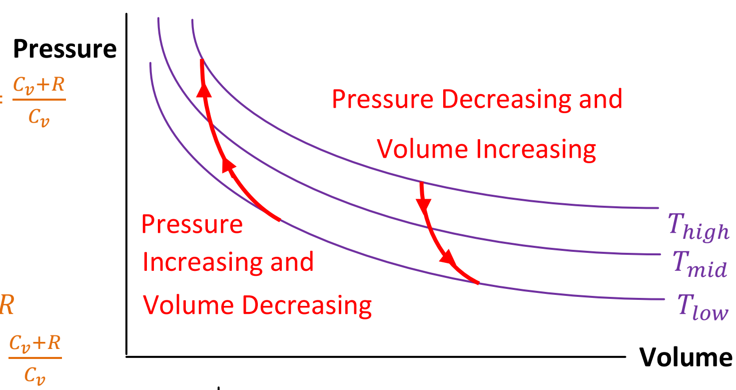
$Heat \quad Q = 0$

$Work \quad W = n\,C_v\,(T_{final} - T_{initial})$

$Entropy \quad \Delta S = 0$

$monatomic\ gas \quad C_v = \frac{3}{2}R \qquad diatomic\ gas \quad C_v = \frac{5}{2}R$

$polyatomic\ gas \quad C_v = 3R \qquad C_p = C_v + R \qquad \gamma = \frac{C_p}{C_v} = \frac{C_v+R}{C_v}$



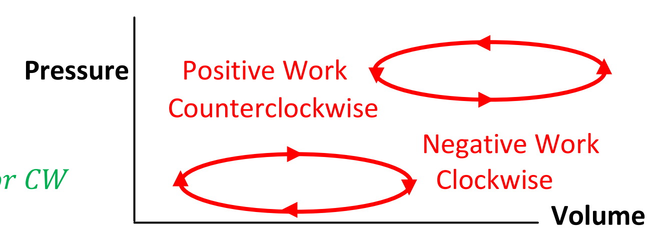
Full Cycle $p_{final} = p_{initial} \qquad V_{final} = V_{initial} \qquad T_{final} = T_{initial}$

$Internal\ Energy \quad \Delta E = 0$

$Heat \quad Q = -W = -Area\ Enclosed\ by\ Cycle\ Curve$

$Work \quad W = Area\ Enclosed\ by\ Cycle\ Curve\ +for\ CCW\ \ -for\ CW$

$Entropy \quad \Delta S = \sum \Delta S_{each\ path\ of\ the\ entire\ cycle}$



Engines and Efficiency, Refrigerators and Coefficient of Performance

Engines and Refrigerators use Thermodynamic Laws to convert between thermal energy and mechanical work energy.

Engines and Efficiency

An Engine is any device that draws heat energy Q_H from a high temperature T_H reservoir, converts only a part of that energy into useful work W, and dumps the remaining energy Q_L into the low temperature T_L reservoir as heat exhaust.

Carnot Engine

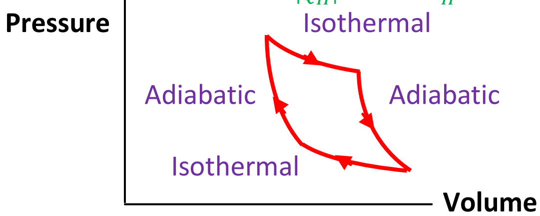
Uses only Adiabatic and Isothermal Paths

$$Work \quad W_{by} = -W = |Q_H| - |Q_L|$$

$$Entropy \quad \Delta S = \Delta S_H + \Delta S_L = \frac{|Q_H|}{T_H} - \frac{|Q_L|}{T_L}$$

$$Efficiency \quad \epsilon_c = \frac{Mechanical\ Work}{Input\ Heat\ Energy} = \frac{|W|}{|Q_H|}$$

$$\epsilon_c = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{T_L}{T_H}$$



Real Engine

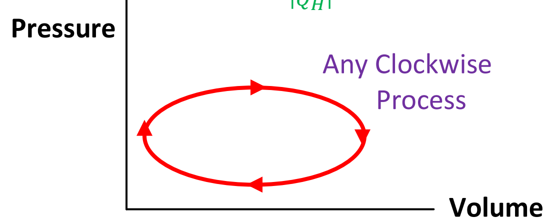
Uses any Paths to complete cycle

$$Work \quad W_{by} = -W = |Q_H| - |Q_L|$$

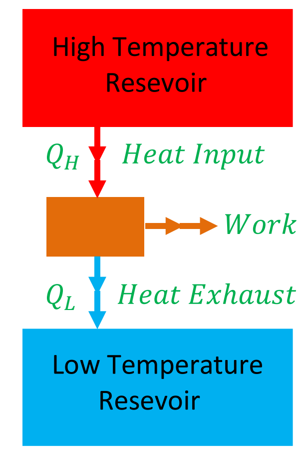
$$Entropy \quad \Delta S = \sum \Delta S_{each\ path\ of\ entire\ cycle}$$

$$Efficiency \quad \epsilon = \frac{Mechanical\ Work}{Input\ Heat\ Energy} = \frac{|W|}{|Q_H|}$$

$$\epsilon = 1 - \frac{|Q_L|}{|Q_H|} \qquad \epsilon < \epsilon_c$$



Engine Diagram



Refrigerators and Coefficient of Performance

A Refrigerator is any device that draws heat energy Q_L from a low temperature T_L reservoir by performing an amount of work W on the system, and dumps the remaining energy Q_H into the high temperature T_H reservoir as heat exhaust.

Carnot Refrigerator

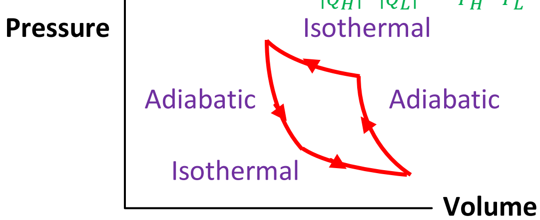
Uses only Adiabatic and Isothermal Paths

$$Work \quad W = |Q_H| - |Q_L|$$

$$Entropy \quad \Delta S = \Delta S_H + \Delta S_L = \frac{|Q_H|}{T_H} - \frac{|Q_L|}{T_L}$$

$$Coefficient\ of\ Performance \quad K_c = \frac{|Q_L|}{|W|}$$

$$K_c = \frac{|Q_L|}{|Q_H| - |Q_L|} = \frac{T_L}{T_H - T_L}$$



Real Refrigerator

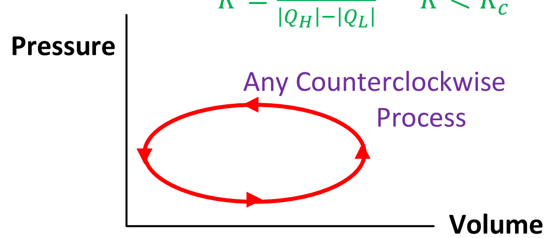
Uses any Paths to complete cycle

$$Work \quad W = |Q_H| - |Q_L|$$

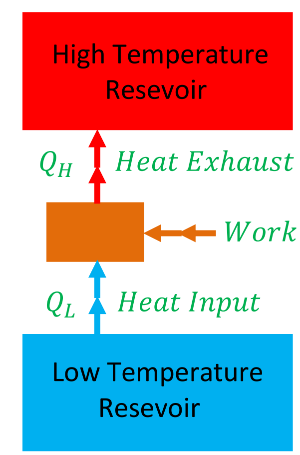
$$Entropy \quad \Delta S = \sum \Delta S_{each\ path\ of\ entire\ cycle}$$

$$Coefficient\ of\ Performance \quad K = \frac{|Q_L|}{|W|}$$

$$K = \frac{|Q_L|}{|Q_H| - |Q_L|} \qquad K < K_c$$



Refrigerator Diagram



Heat Transfer Methods

Heat is the transfer of thermal energy and can be accomplished by methods of Convection, Conduction, and Radiation.

Convection

Convection is transfer of heat energy by physical motion of fluid particles over a temperature difference.

Radiation

Radiation is transfer of heat energy by emission of light particles or photons from a hot body.

$$P_{radiation} = \sigma\, \varepsilon\, A\, T^4$$

$$\sigma = 5.6703 \times 10^{-8}\ W/m^2 \cdot K^4 \qquad \varepsilon = emissivity$$

$$A = cross\ sectional\ area \quad T = temperature\ in\ K$$

Conduction

Conduction is a transfer of vibrational energy between adjacent atoms over a temperature difference.

$$P_{conduction\ one\ object} = \frac{Q}{t} = \frac{k\, A\, (T_{High} - T_{Low})}{L}$$

$$P_{conduction\ multiple\ objects} = \frac{Q}{t} = \frac{A\,(T_{High} - T_{Low})}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}}$$

$$k = thermal\ conductivity \quad L = slab\ length$$

$$A = cross\ sectional\ area \quad T = temperature\ in\ K$$

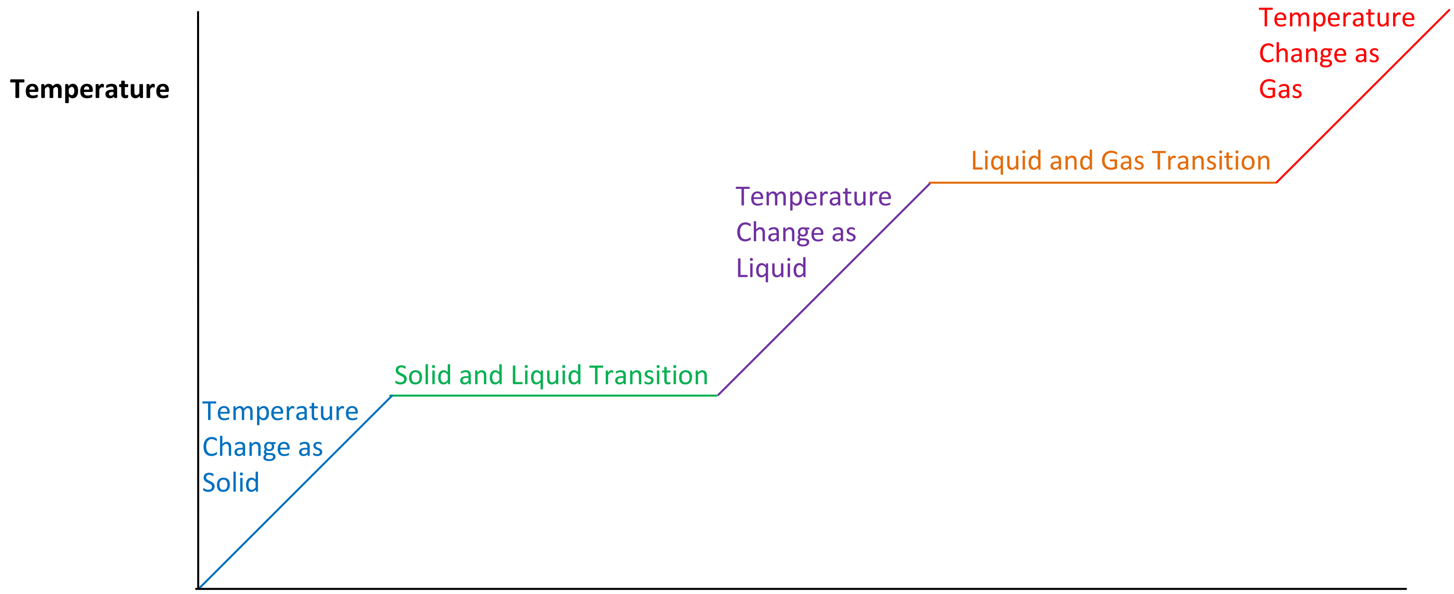
Heat, Temperature Change, Phase Change for Solids-Liquids-Gases and the Heating Curve

Heating Curve displays Temperature Change within the Phases (Solid, Liquid, or Gas) as a diagonal line with temperature change directly proportional to the heat input and a Phase Transition as a horizontal line at a constant temperature.

$$Object\ Heat\ Capacity\ \ C = \frac{q}{\Delta T} \qquad Mass\ Heat\ Capacity\ \ c = \frac{q}{m\,\Delta T} \qquad Molar\ Heat\ Capacity\ \ c = \frac{q}{n\,\Delta T}$$

$$Heat\ of\ single\ substance\ \ q_{total} = \sum q_{each\ temperature\ change} + q_{each\ phase\ change} = \sum m_i\, c_i\, \Delta\, T_i + m_i\, L_i$$

$$Heat\ of\ multiple\ substances\ \ q_{total} = 0 = \sum q_{each\ temperature\ change} + q_{each\ phase\ change} = \sum m_i\, c_i\, \Delta\, T_i + m_i\, L_i$$



←←← **Heat Removed Exothermic to the Left** **or** **Heat Added Endothermic to the Right** →→→

Important colored paths on the Heating Curve above are the Temperature Changes or Phase Transitions as follows:

Blue Path Solid Phase Temperature Change

The blue path is the solid phase temperature increase as heat is input or temperature decrease as heat is output. The heat input q and temperature change ΔT are related through the mass m or moles n and the solid specific heat c_s by

$$mass\ version \quad q = m\, c_s\, \Delta T_s \quad slope\ of\ line = \frac{1}{m\, c_s} \qquad mole\ version \quad q = n\, c_s\, \Delta T_s \quad slope\ of\ line = \frac{1}{n\, c_s}$$

Green Path Solid and Liquid Phase Transition

The green path is the solid and liquid phase transition at constant temperature known as freezing point temperature T_f. The heat input q of this phase transition is related to the heat of fusion L_{fus} and the number of moles n by

$$mass \quad q = \pm m\, L_{fus} \quad + for\ forward, - for\ reverse \qquad mole \quad q = \pm n\, L_{fus} \quad + for\ forward, - for\ reverse$$

Purple Path Liquid Phase Temperature Change

The purple path is the liquid phase temperature increase as heat is input or temperature decrease as heat is output. The heat input q and temperature change ΔT are related through the mass m or moles n and the liquid specific heat c_l by

$$mass\ version \quad q = m\, c_l\, \Delta T_l \quad slope\ of\ line = \frac{1}{m\, c_l} \qquad mole\ version \quad q = n\, c_l\, \Delta T_l \quad slope\ of\ line = \frac{1}{n\, c_l}$$

Orange Path Liquid and Gas Phase Transition

The orange path is the liquid and gas phase transition at constant temperature known as boiling point temperature T_b. The heat input q of this phase transition is related to the heat of vaporization L_{vap} and the number of moles n by

$$mass \quad q = \pm m\, L_{vap} \quad + for\ forward, - for\ reverse \qquad mole \quad q = \pm n\, L_{vap} \quad + for\ forward, - for\ reverse$$

Red Path Gas Phase Temperature Change

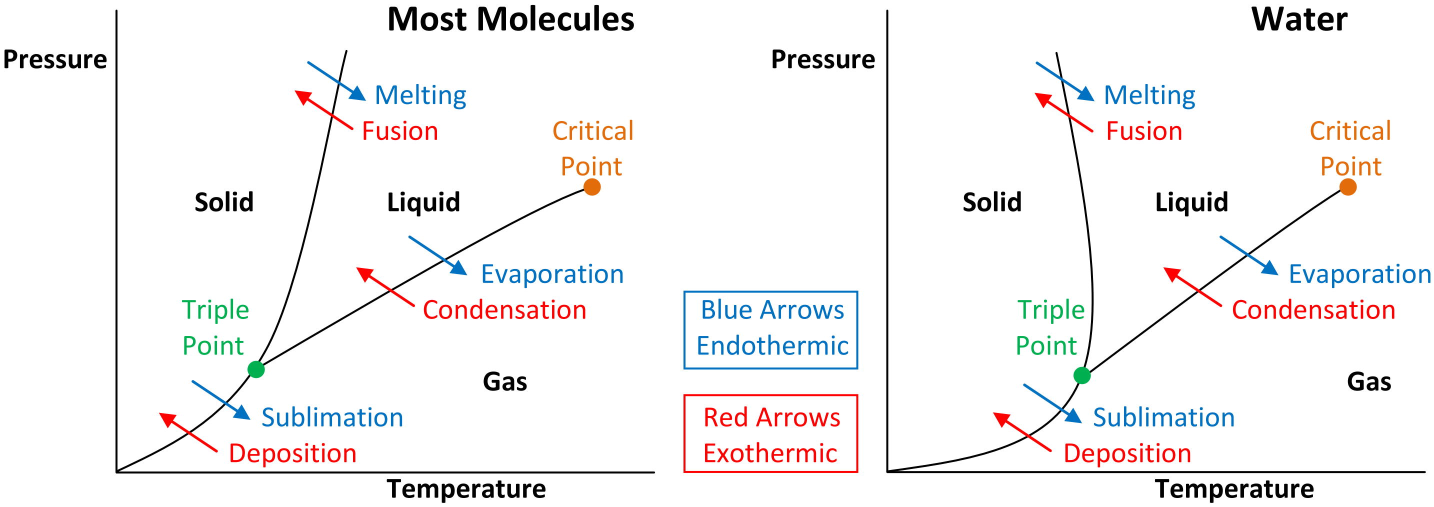
The red path is the gas phase temperature increase as heat is input or temperature decrease as heat is output. The heat input q and temperature change ΔT are related through the mass m or moles n and the gas specific heat c_g by

$$mass\ version \quad q = m\, c_g\, \Delta T_g \quad slope\ of\ line = \frac{1}{m\, c_g} \qquad mole\ version \quad q = n\, c_g\, \Delta T_g \quad slope\ of\ line = \frac{1}{n\, c_g}$$

Phase Changes and the Phase Diagram

Phase Diagram is a graph of the pressure vs. temperature for the phase boundaries between solid, liquid, and gas states.
Solid (s) has strong intermolecular forces and retains its shape regardless of the container that it exists in.
Liquid (l) has moderate intermolecular forces and has a shape that retains its volume but fills a container to a fixed level.
Gas (g) has weak intermolecular forces and has a shape that matches whatever container that it always completely fills.

On a Phase Diagram any horizontal path is a constant pressure path and any vertical path is a constant temperature path



Important Points on the Phase Diagrams above are special pressures and temperatures as follows:

1. **Triple Point** The green point is the Triple Point, exact pressure and temperature values where all three phases of solid (s), liquid (l), and gas (g) exist in equilibrium. This point is fixed for a substance and is used for calibration.
2. **Critical Point** The orange point is the Critical Point, exact pressure and temperature values at which there no longer exists a clear difference between the liquid and gas phases of the substance. Above this pressure and above this temperature the substance becomes a supercritical fluid with properties of both a liquid and a gas.

Important Arrow Directions on the Phase Diagrams above are the Phase Transitions as follows:

1. **Melting or Liquification** The blue arrow indicates the endothermic transition from solid to liquid.
   For most molecules to reach this transition requires either an increase in temperature or a decrease in pressure.
   For water to reach this transition requires either an increase in temperature or an increase in pressure.
   The energy input required for this transition is known as the Heat of Liquification $\Delta H_{liq} = \Delta H_{fus}$
2. **Fusion or Solidification** The red arrow indicates the exothermic transition from liquid to solid.
   For most molecules to reach this transition requires either a decrease in temperature or an increase in pressure.
   For water to reach this transition requires either a decrease in temperature or a decrease in pressure.
   The energy output created by this transition is known as the Heat of Solidification $\Delta H_{sol} = -\Delta H_{fus}$
3. **Evaporation or Vaporization** The blue arrow indicates the endothermic transition from liquid to gas.
   For all molecules to reach this transition requires either an increase in temperature or a decrease in pressure.
   The energy input required for this transition is known as the Heat of Evaporization $\Delta H_{evap} = \Delta H_{vap}$
4. **Condensation** The red arrow indicates the exothermic transition from gas to liquid.
   For all molecules to reach this transition requires either a decrease in temperature or an increase in pressure.
   The energy output created by this transition is known as the negative Heat of Vaporization $\Delta H_{cond} = -\Delta H_{vap}$
5. **Sublimation** The blue arrow indicates the endothermic transition from solid to gas.
   For all molecules to reach this transition requires either an increase in temperature or a decrease in pressure.
   The energy input required by this transition is known as the Heat of Sublimation $\Delta H_{sub} = \Delta H_{fus} + \Delta H_{vap}$
6. **Deposition** The red arrow indicates the exothermic transition from gas to solid.
   For all molecules to reach this transition requires either a decrease in temperature or an increase in pressure.
   The energy output created by this transition is known as the Heat of Deposition $\Delta H_{dep} = -\Delta H_{fus} - \Delta H_{vap}$