GradePeak
private instruction

PHYS 1

CONCEPT PACKET
Complete

Written by Jeremy Robinson, Head Instructor

Need Help?

Online Private Instruction
Anytime, Anywhere
Any High School!
Any College!

Individual or Small Group

Visit www.gradepeak.com

Call or Text 814.867.1777

Vectors

Vectors differ from Scalars in they have magnitude and direction. Position, Velocity, Acceleration, and Force are Vectors.

Two Dimensional Vector Component Form or Vector Magnitude Direction Form

The Vector Component Form states a vector by listing its amount in each of the coordinate axis directions.

$$\vec{v} = (v_x)i + (v_y)j$$

v_x is the amount along i or the x axis, v_y is the amount along j or the y axis

The Vector Magnitude Direction Form states a vector by listing its amount and angle to the coordinate axis directions.

$$\vec{v} = \textit{magnitude } v \textit{ at an angle of } \theta \textit{ to one of the coordinate axes}$$

Conversion from Two Dimensional Component Form to Magnitude Direction Form

The Components of a Vector are perpendicular and form a right triangle, leaving the Magnitude and Direction relations

$$v = |\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{(v_x)^2 + (v_y)^2} \qquad \theta_{reference} = \tan^{-1}\left|\frac{v_y}{v_x}\right|$$

$\theta = \theta_{reference}$ for a Vector with $Q1$ Orientation

$\theta = 180^\circ - \theta_{reference}$ for a Vector with $Q2$ Orientation

$\theta = 180^\circ + \theta_{reference}$ for a Vector with $Q3$ Orientation

$\theta = 360^\circ - \theta_{reference}$ *or* $\theta = -\theta_{reference}$ for a Vector with $Q4$ Orientation

$Q1$ Orientation for a Vector that points towards the first quadrant, it will have a $+x$ component and a $+y$ component

$Q2$ Orientation for a Vector that points towards the second quadrant, it will have a $-x$ component and a $+y$ component

$Q3$ Orientation for a Vector that points towards the third quadrant, it will have a $-x$ component and a $-y$ component

$Q4$ Orientation for a Vector that points towards the fourth quadrant, it will have a $+x$ component and a $-y$ component

Conversion from Two Dimensional Magnitude Direction Form to Component Form

If the angle θ is to nearest x axis	If the angle θ is to nearest y axis	If the angle θ is to the $+x$ axis
$v_x = \pm v\cos\theta$	$v_x = \pm v\sin\theta$	$v_x = v\cos\theta$
$v_y = \pm v\sin\theta$	$v_y = \pm v\cos\theta$	$v_y = v\sin\theta$
Sign determined by Q Orientation	Sign determined by Q Orientation	Sign is automatically determined

Three Dimensional Vector Component Form and Magnitude

The Vector Component Form states a vector by listing its amount in each of the coordinate axis directions.

$$\vec{v} = (v_x)i + (v_y)j + (v_z)k$$

v_x is the amount along i or the x axis, v_y is the amount along j or the y axis, v_z is the amount along k or the z axis

The Components of a Vector are perpendicular and form a right triangle, leaving the Magnitude relation

$$v = |\vec{v}| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$$

Unit Vector and Vector Scaling

A Unit Vector $\vec{w}$ has a magnitude of exactly one while still being in the same direction as some other vector $\vec{v}$.

$$\vec{w} = \frac{\vec{v}}{|\vec{v}|} = \frac{\vec{v}}{\sqrt{\vec{v} \cdot \vec{v}}} \qquad \textit{so that vector } \vec{w} \textit{ has the direction of } \vec{v} \textit{ and a magnitude of } |\vec{w}| = 1$$

A Vector $\vec{u}$ of magnitude k but in the direction of some other vector $\vec{v}$ can be calculated through a Unit Vector $\vec{w}$ as

$$\vec{u} = k\,\vec{w} = k\,\frac{\vec{v}}{|\vec{v}|} = k\,\frac{\vec{v}}{\sqrt{\vec{v} \cdot \vec{v}}} \qquad \textit{so that vector } \vec{u} \textit{ has the direction of } \vec{v} \textit{ and a magnitude of } |\vec{u}| = k$$

Vector Products

Dot Product or Scalar Product produces a scalar quantity that is used for vector components and vector projections

$$\vec{u} \cdot \vec{v} = \left[(u_x)i + (u_y)j + (u_z)k\right] \cdot \left[(v_x)i + (v_y)j + (v_z)k\right] = u_x v_x + u_y v_y + u_z v_z$$

$$\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}|\cos\theta \qquad \textit{where } \theta \textit{ is the tail to tail angle between } \vec{u} \textit{ and } \vec{v}$$

Cross Product or Vector Product produces a vector quantity that is simultaneously perpendicular to each original vector

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = \begin{vmatrix} i & \boxtimes & \boxtimes \\ \boxtimes & u_y & u_z \\ \boxtimes & v_y & v_z \end{vmatrix} - \begin{vmatrix} \boxtimes & j & \boxtimes \\ u_x & \boxtimes & u_z \\ v_x & \boxtimes & v_z \end{vmatrix} + \begin{vmatrix} \boxtimes & \boxtimes & k \\ u_x & u_y & \boxtimes \\ v_x & v_y & \boxtimes \end{vmatrix} = i\begin{vmatrix} u_y & u_z \\ v_y & v_z \end{vmatrix} - j\begin{vmatrix} u_x & u_z \\ v_x & v_z \end{vmatrix} + k\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

$$\vec{u} \times \vec{v} = \left[(u_x)i + (u_y)j + (u_z)k\right] \times \left[(v_x)i + (v_y)j + (v_z)k\right] = (u_y v_z - u_z v_y)i + (u_z v_x - u_x v_z)j + (u_x v_y - u_y v_x)k$$

$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\sin\theta \qquad \textit{where } \theta \textit{ is the tail to tail angle between } \vec{u} \textit{ and } \vec{v}$$

Kinematics Formulas

The Kinematics Formulas relate the position, velocity, and acceleration of an object through the time parameter t.

True Vector Kinematics Formulas

Instant Velocity $\vec{v}$ or ω and Position $\vec{r}$ or θ Relations

$\vec{v}(t) = \frac{d\vec{r}}{dt}$ $\quad$ $\omega(t) = \frac{d\theta}{dt}$

$\vec{r_f}(t) - \vec{r_i}(t) = \int \vec{v}(t)\, dt$ $\quad$ $\theta_f(t) - \theta_i(t) = \int \omega(t)\, dt$

On $\vec{r}$ or θ versus t graph, $\vec{v}$ or ω is slope of tangent line

On $\vec{v}$ or ω versus t graph, $\vec{r}$ or θ is area under the curve

Average Velocity and Position Difference Relations

$\vec{v}_{avg}(t) = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r_f}(t) - \vec{r_i}(t)}{\Delta t}$ $\quad$ $\omega_{avg}(t) = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f(t) - \theta_i(t)}{\Delta t}$

Instant Acceleration $\vec{a}$ or α and Velocity $\vec{v}$ or ω Relations

$\vec{a}(t) = \frac{d\vec{v}}{dt}$ $\quad$ $\alpha(t) = \frac{d\omega}{dt}$

$\vec{v_f}(t) - \vec{v_i}(t) = \int \vec{a}(t)\, dt$ $\quad$ $\omega_f(t) - \omega_i(t) = \int \alpha(t)\, dt$

On $\vec{v}$ or ω versus t graph, $\vec{a}$ or α is slope of tangent line

On $\vec{a}$ or α versus t graph, $\vec{v}$ or ω is area under the curve

Average Acceleration and Velocity Difference Relations

$\vec{a}_{avg}(t) = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v_f}(t) - \vec{v_i}(t)}{\Delta t}$ $\quad$ $\alpha_{avg}(t) = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f(t) - \omega_i(t)}{\Delta t}$

Constant Acceleration $\vec{a}$ Vector Kinematics Formulas

vector direction

$\vec{v}(t) = \vec{v}_0 + \vec{a}t$

$\vec{r}(t) = \vec{r}_0 + \vec{v}_0\, t + \frac{1}{2}\, \vec{a}\, t^2$

$\vec{r}_0$ initial vector position at time $t = 0$

$\vec{r}$ final vector position at any time t

$\vec{v}_0$ initial vector velocity at time $t = 0$

$\vec{v}$ final vector velocity at any time t

Constant Acceleration a_x, a_y, α Component Kinematics Formulas

x direction

$v_x(t) = v_{x0} + a_x\, t$

$x(t) = x_0 + v_{x0}\, t + \frac{1}{2} a_x\, t^2$

$v_x^2 = v_{x0}^2 + 2\, a_x\, (x - x_0)$

x_0 initial x position at time $t = 0$

x final x position at any time t

v_{x0} initial x velocity at time $t = 0$

v_x final x velocity at any time t

y direction

$v_y(t) = v_{y0} + a_y\, t$

$y(t) = y_0 + v_{y0}\, t + \frac{1}{2}\, a_y\, t^2$

$v_y^2 = v_{y0}^2 + 2\, a_y\, (y - y_0)$

y_0 initial y position at time $t = 0$

y final y position at any time t

v_{y0} initial y velocity at time $t = 0$

v_y final y velocity at any time t

θ direction

$\omega(t) = \omega_o + \alpha\, t$

$\theta(t) = \theta_o + \omega_o\, t + \frac{1}{2} \alpha\, t^2$

$\omega^2 = \omega_o^2 + 2\, \alpha\, (\theta - \theta_o)$

θ_o initial θ position at time $t = 0$

θ final θ position at any time t

ω_o initial θ velocity at time $t = 0$

ω final θ velocity at any time t

The Kinematics Formulas in the different directions are completely independent except for the time t which is the same.

If the object is moving with an initial velocity v_0 at an initial angle θ_0 with respect to the x axis

$$v_{x0} = v_0\, \cos\theta_0$$
$$v_{y0} = v_0\, \sin\theta_0$$

If the object is moving with an initial velocity v_0 at an initial angle θ_0 with respect to the y axis

$$v_{x0} = v_0\, \sin\theta_0$$
$$v_{y0} = v_0\, \cos\theta_0$$

Projectile Motion

For projectile motion, the following two conditions exist:

1. The acceleration in the x-direction is equal to zero $a_x = 0$
2. The acceleration in the y-direction is equal to the gravitational acceleration $a_y = -g = -9.81\, \frac{m}{s^2} = -32.2\, \frac{ft}{s^2}$

Range Equation $R = \frac{{v_0}^2\, \sin 2\theta_0}{g}$ When $y = y_0$ **Path Equation** $y = (\tan\theta_0)x - \frac{g\, x^2}{2(v_0 \cos\theta_0)^2}$ When $y_0 = 0$ and $x_0 = 0$

When is the particle at rest? Set the velocity function equal to zero $v(t) = 0$ and solve for t.

When or what is the maximum/minimum position or height? Set the velocity function equal to zero $v(t) = 0$ and solve for t. Plug this value t into the position function $s(t)$ to find the maximum or minimum position or height.

When or what is the maximum/minimum velocity or speed? Set the acceleration function equal to zero $a(t) = 0$ and solve for t. Plug this value t into the velocity function $v(t)$ to find the maximum or minimum velocity.

Conversions between Linear and Angular Quantities

In rotational motion all points on an object will have equal angular quantities θ, ω, α but unequal linear quantities s, v, a

Tangential Linear Distance s and Angular Distance θ

$$s = r\theta$$

Tangential Linear Velocity v and Angular Velocity ω

$$v = r\omega$$

Tangential Linear Acceleration a and Angular Acceleration α

$$a = r\alpha$$

Relative Kinematic Relations

Relative Kinematic Relations equate the kinematic quantities of object A with frame C to object or frame B with frame C.

$$\vec{r_{AC}} = \vec{r_{AB}} + \vec{r_{BC}} \qquad \vec{v_{AC}} = \vec{v_{AB}} + \vec{v_{BC}} \qquad \vec{a_{AC}} = \vec{a_{AB}} + \vec{a_{BC}}$$

Newtons Laws

Newtons Laws are the rules and equations involving the relation of forces and the kinematic quantity acceleration.

Newtons First Law

Newtons First Law is the definition behind the concept of Force. It states that an object in motion or at rest will remain in motion or at rest unless acted on by a Force. A Force produces an acceleration which in turn changes the velocity.

Newtons Second Law Vector Form

vector direction

$\sum \vec{F} = m\,\vec{a}$

Sign Convention for each term in the sum $\sum \vec{F}$

- F_x is positive for a $+x\ axis$ directed force
- F_x is negative for a $-x\ axis$ directed force

Component in the sum $\sum \vec{F}$

If θ is the angle to the nearest x axis

$F_x = F\cos\theta$

If θ is the angle to the nearest y axis

$F_x = F\sin\theta$

Sign Convention for each term in the sum $\sum \vec{F}$

- F_y is positive for a $+y\ axis$ directed force
- F_y is negative for a $-y\ axis$ directed force

Component in the sum $\sum \vec{F}$

If θ is the angle to the nearest x axis

$F_y = F\sin\theta$

If θ is the angle to the nearest y axis

$F_y = F\cos\theta$

Newtons Second Law Component Form

x direction

$\sum F_x = m\,a_x$

Sign Convention for each term in the sum $\sum F_x$

- F_x is positive for a $+x\ axis$ directed force
- F_x is negative for a $-x\ axis$ directed force

Component in the sum $\sum F_x$

If θ is the angle to the nearest x axis

$F_x = F\cos\theta$

If θ is the angle to the nearest y axis

$F_x = F\sin\theta$

y direction

$\sum F_y = m\,a_y$

Sign Convention for each term in the sum $\sum F_y$

- F_y is positive for a $+y\ axis$ directed force
- F_y is negative for a $-y\ axis$ directed force

Component in the sum $\sum F_y$

If θ is the angle to the nearest x axis

$F_y = F\sin\theta$

If θ is the angle to the nearest y axis

$F_y = F\cos\theta$

θ direction

$\sum \tau_z = I\,\alpha_z$

Sign Convention for each term in the sum $\sum \tau_z$

- τ_z is positive for a $+z\ axis$ directed torque
- τ_z is negative for a $-z\ axis$ directed torque

Component in the sum $\sum \tau_z$

$\vec{\tau} = \vec{r} \times \vec{F}\ \ vector \qquad \tau = r\,F\sin\theta = (r\,F)_{perp}\ \ scalar$

θ is the angle between r and F

r_{perp} is the distance component perpendicular to F

F_{perp} is the force component perpendicular to r

$\bar{I}_{point\ mass\ or\ hoop\ or\ shell\ cylinder} = m\,R^2$

$\bar{I}_{disk\ or\ solid\ cylinder} = \frac{1}{2} m\,R^2$

$\bar{I}_{solid\ sphere} = \frac{2}{5} m\,R^2 \qquad \bar{I}_{shell\ sphere} = \frac{2}{3} m\,R^2$

$\bar{I}_{rod\ about\ center} = \frac{1}{12} m\,L^2 \qquad I_{rod\ about\ end} = \frac{1}{3} m\,L^2$

Parallel Axis Theorem $\quad I = \bar{I} + m\,h^2$

I is the new moment of inertia

$\bar{I}$ is the center of mass moment of inertia

m is the mass of the object

h is the distance between the center of mass of the object and the new axis of rotation

Circular Motion

For circular motion, always define a radial axis r from the object in motion towards the center of the circle. The centripetal direction is along this radial axis r and the acceleration in this direction always has the magnitude

$$a_r = \frac{v^2}{r}$$

Centripetal acceleration a_r is the acceleration resulting from a net force toward the center of the circle that enables an object to move in a circle. It can be related to the force toward the center of the circle through Newtons Second Law.

Forces

Weight or Gravitational Force

Symbol	Formula	Direction
W	$W = mg$ where m is the mass of the object and g is the gravitational acceleration $g = 9.81\ \frac{m}{s^2} = 32.2\ \frac{ft}{s^2}$	Always towards the center of the Earth, which will often be downward in most Free Body Diagrams

Tension Force

Symbol	Formula	Direction
T	*Newtons Second Law* where T may be one of the unknown forces and can be solved by relating to the other forces	Always along the rope or cable that is producing the force and away from the object that it is attached to

Normal Force

Symbol	Formula	Direction
N	*Newtons Second Law* where N may be one of the unknown forces and can be solved by relating to the other forces	Always perpendicular and tangent to the surfaces of contact between the two objects

Static Friction Force

Symbol	Formula	Direction
f_s	$f_s \leq \mu_s\, N$ where μ_s is the coefficient of static friction at the surface of contact and N is the normal force at the same surface of contact	Always along the surface of contact and opposite the impending sliding motion direction of the object

1. $f_s < \mu_s\, N$
   In this the case the object is not sliding, nor is it near impending sliding.
2. $f_s = \mu_s\, N$
   In this the case the object is impending sliding
3. $f_s > \mu_s\, N$
   The object is sliding, and the friction force will be the kinetic friction force and not the static friction force.

Kinetic Friction Force

Symbol	Formula	Direction
f_k	$f_k = \mu_k\, N$ where μ_k is the coefficient of kinetic friction at the surface of contact and N is the normal force at the same surface of contact	Always along the surface of contact and opposite the sliding motion direction of the object

Spring Force

Symbol	Formula	Direction
F_{spring}	$F_{spring} = k\ s$ where k is the spring constant of the particular spring and s is the stretched or compressed distance of the spring $s = L_f - L_i$	Always along the spring and away from the object if the spring is stretched or towards the object if the spring is compressed

Newtons Third Law

Newtons Third Law is the description of the Force interaction between two objects. It states that for a Force on object 1 due to object 2 there will always be an equal magnitude but oppositely directed Force on object 2 due to object 1. Every force will always have its reaction force also present, but the reaction force is always acting on the other object.

$$\vec{F}_{on\ object\ 1\ due\ to\ object\ 2} = -\vec{F}_{on\ object\ 2\ due\ to\ object\ 1}$$

Work and Power

Work is the energy supplied to or taken away by a force and power is the time rate at which that energy is generated.

Linear Force Work

Work is the energy supplied to (positive) or taken away from (negative) an object by a force as an object displaces by a distance s. Any force components parallel to the direction of motion will supply positive work and any force components anitparallel to the direction of motion will supply negative work. Any force components perpendicular to the direction of motion will do zero work. For circular motion the direction of motion is tangent to the curve and all force components in the centripetal direction do zero work. The definition of linear work W is with an integral for a variable force vector $\vec{F}$

$$W = \int \vec{F} \cdot d\vec{s} = \int F \cos\theta \, ds \qquad\qquad W = \vec{F} \cdot \vec{s} = F\, d \cos\theta \quad \text{if } \vec{F} \text{ is constant over linear path } \vec{s}$$

The forces doing work on an object can be one or more of the six fundamental forces:

1. The Gravitational Force or Weight
   $F_g = W = mg$
2. The Normal Force
   $F_n = N$
3. The Tension Force
   $F_t = T$
4. The Static Friction Force
   $f_s \leq \mu_s F_n$
5. The Kinetic Friction Force
   $f_k = \mu_k F_n$
6. The Spring Force
   $F_{spring} = k\, s$

Angular Force or Torque Work

The definition of angular work W is through an integral to allow for a variable torque τ

$$W = \int \tau \, d\theta \qquad\qquad \mathrm{W} = \tau\, \theta \quad \text{if } \tau \text{ is constant over angular path } \theta$$

Power

The definition of power P is the energy or work rate and is through a derivative to allow for a variable force or torque

$$P = \frac{dW}{dt} = \frac{dU}{dt} = \frac{dK}{dt} \qquad\qquad P_{average} = \frac{\Delta W}{\Delta t} = \frac{\Delta U}{\Delta t} = \frac{\Delta K}{\Delta t}$$

$P = \vec{F} \cdot \vec{v} = F\, v \cos\theta$ — *For a force F on an object with a linear speed v and angle θ between F and v*

$P = \tau\, \omega \cos\theta$ — *For a torque τ on an object with a angular speed ω and angle θ between τ and ω*

Conservative Forces

Two of the six fundamental forces have a property known as Conservative, the Gravitational Force and the Spring Force.

- The Gravitational Force or Weight
  $F_g = W = mg$
- The Spring Force
  $F_s = k\, s$

Conservative Forces do not cause a gain or loss of the total energy to a system, but rather store it into a reusable form of energy known as Potential Energy. The work done by a Conservative Force is independent of the path taken by the object, but rather depends only on the displacement or the difference between the initial point and final point.

- Work done by the Gravitational Force or Weight
  $W = -\Delta U_g = -m\, g\, (y_{final} - y_{initial})$
- Work done by the Spring Force
  $W = -\Delta U_s = -\frac{1}{2}\, k\, (s_{final}^2 - s_{initial}^2)$

Nonconservative Forces

Four of the six fundamental forces are Nonconservative, the Normal Force, the Tension Force, and both Friction Forces.

- Work done by the Normal Force
  $W = N\, d \cos\theta$
- Work done by the Tension Force
  $W = T\, d \cos\theta$
- Work done by the Static Friction Force
  $W = f_s\, d \cos\theta$
- Work done by the Kinetic Friction Force
  $W = f_k\, d \cos\theta$

Nonconservative Forces can cause a loss or a gain of energy to a system, and their work will be dependent on the path or the total amount of distance travelled by the object in displacing from an initial point to a final point.

Work and Energy Conservation

Single Object Work Energy Conservation Equation

The sum of the Work terms created by each and every force acting on an object will be equal to the change in the Kinetic Energy of the object over the distance path. This relationship is known as the Work and Energy Conservation Equation

$$\sum W = \Delta K$$

The Work and Energy Conservation Equation in a form where the Conservative Force Work terms (negative changes in potential energies) and the Nonconservative Force Work terms have been separated on each side of the equation

$$\sum W_{nc} = \Delta K + \Delta U_g + \Delta U_s$$

This can also be expanded by the definition of Work $W = Fd\cos\theta$, translational Kinetic Energy $K = \frac{1}{2}mv^2$, Rotational Kinetic Energy $K = \frac{1}{2}\,I\omega^2$, Gravitational Potential Energy $U_g = mgy$, and Spring Potential Energy $U_s = \frac{1}{2}ks^2$ into

$$\sum F_{nc}\,d\cos\theta = \frac{1}{2}\,m\left(v_{final}^2 - v_{initial}^2\right) + \frac{1}{2}\,I\left(\omega_{final}^2 - \omega_{initial}^2\right) + m\,g\left(y_{final} - y_{initial}\right) + \frac{1}{2}\,k\left(s_{final}^2 - s_{initial}^2\right)$$

Rolling and Pivoted Motion Relations

If the object or system is Rolling or Pivoted, the translational quantities and angular quantities are related by

Tangential Linear Distance s and Angular Distance θ

$s = r\,\theta$

Tangential Linear Velocity v and Angular Velocity ω

$v = r\,\omega$

Tangential Linear Acceleration a and Angular Acceleration α

$a = r\,\alpha$

Where r is the distance between the center of mass and the rolling contact point or the pivot point

Center of Mass Motion

The center of mass is a very important point for an object. It is defined as the point where there is an equal mass contribution in all directions around the point. The center of mass point will be the location of a free object that will follow all four dynamics equation sets:

- Kinematics
- Newton's Laws
- Work-Energy Conservation
- Impulse-Momentum Conservation

An object or system that is both translating (straight line motion) and rotating (curvilinear motion) will have the rotation axis centered around one of three points:

1. If the object or system is hinged or pivoted, it will rotate around the hinge or pivot point.
2. If the object or system is rolling, it will rotate around the center of the object.
3. If the object or system is free, that is not rolling, hinged, or pivoted, it will rotate around the center of mass.

Center of Mass–Position

$$\bar{x} = \frac{\sum m_n\,\overline{x_n}}{\sum m_n}$$
$$\bar{y} = \frac{\sum m_n\,\overline{y_n}}{\sum m_n}$$
$$\bar{z} = \frac{\sum m_n\,\overline{z_n}}{\sum m_n}$$

Center of Mass –Velocity

$$\overline{v_x} = \frac{\sum m_n\,\overline{v_{nx}}}{\sum m_n}$$
$$\overline{v_y} = \frac{\sum m_n\,\overline{v_{ny}}}{\sum m_n}$$
$$\overline{v_z} = \frac{\sum m_n\,\overline{v_{nz}}}{\sum m_n}$$

Center of Mass–Acceleration

$$\overline{a_x} = \frac{\sum m_n\,\overline{a_{nx}}}{\sum m_n}$$
$$\overline{a_y} = \frac{\sum m_n\,\overline{a_{ny}}}{\sum m_n}$$
$$\overline{a_z} = \frac{\sum m_n\,\overline{a_{nz}}}{\sum m_n}$$

Rocket Equations

A Rocket is an object that is propelled forward by center of mass motion as fuel is expelled in the opposite direction.

$$F = Thrust = R_{fuel\ mass\ burn\ rate}\,v_{relative} = M\,a$$

$$M_{final} = M_{initial} - M_{fuel\ burned} = M_{initial} - R_{fuel\ mass\ burn\ rate}\,t \quad where \quad M_{fuel\ burned} = R_{fuel\ mass\ burn\ rate}\,t$$

$$v_{final} - v_{initial} = v_{relative}\,\ln\left(\frac{M_{initial}}{M_{final}}\right) = v_{relative}\,\ln\left(\frac{M_{initial}}{M_{initial} - M_{fuel\ burned}}\right)$$

Linear Impulse and Momentum

Linear Momentum

Linear Momentum $\vec{p}$ is a vector value with a direction and is the measurement of the linear velocity of a center of mass $\vec{v}$ of an object or a system of objects, multiplied by the total mass of the object or a system of objects.

$$\vec{p}_{object} = m_{object}\ \vec{v}_{object}$$
$$\vec{p}_{system} = m_{system\ total}\ \vec{v}_{center\ of\ mass\ of\ the\ system}$$

Linear Impulse

Linear Impulse $\vec{J}$ is a vector value with a direction and is the action of a force vector $\vec{F}$ times a time interval t during which that force acts. The object does not need to move a distance over the time interval for an impulse to be created.

$$\vec{J}_{system\ of\ forces} = \int \sum \vec{F}\ dt \quad = \sum \vec{F}\ t\ \ if\ the\ force\ vector\ \vec{F}\ is\ constant$$

Linear Impulse Momentum Conservation

The total momentum vectors $\vec{p}$ of each object in a system of objects is only changed when there is a total impulse vector $\vec{J}$ acting on it, or equivalently when there is a nonzero net force vector $\vec{F}$ acting on it through a time interval t

$$\vec{J}_{system\ of\ forces} = \sum \vec{p}_{final\ each\ object} - \sum \vec{p}_{initial\ each\ object} = \int \sum \vec{F}_{system\ of\ objects}\ dt$$
$$\vec{J}_{system\ of\ forces} = \sum m_{each\ object}\ \vec{v}_{final\ object} - \sum m_{each\ object}\ \vec{v}_{initial\ object} = \int \sum \vec{F}_{system\ of\ objects}\ dt$$

Collisions and Masses Meeting Together, Explosions and Masses Breaking Apart

Two Dimensional Inelastic Collisions

During an inelastic collision there is nonconservative work and energy lost due to thermal energy generation by kinetic friction and deformation of the objects by normal forces. The three equations that govern this situation are

$$\sum m_{each\ object}\ v_{final\ x\ each\ object} - \sum m_{each\ object}\ v_{initial\ x\ each\ object} = 0$$
$$\sum m_{each\ object}\ v_{final\ y\ each\ object} - \sum m_{each\ object}\ v_{initial\ y\ each\ object} = 0$$
$$\sum W_{nc} = \sum \frac{1}{2} m_{each\ object}\ v^2_{final\ each\ object} - \sum \frac{1}{2} m_{each\ object}\ v^2_{initial\ each\ object}$$

Two Dimensional Completely Inelastic Collisions

During a completely inelastic collision the two objects stick together as a result of the collision and there is a maximum nonconservative work and maximum energy lost. The three equations that govern this situation are

$$\sum m_{each\ object}\ v_{final\ x\ each\ object} - \sum m_{each\ object}\ v_{initial\ x\ each\ object} = 0$$
$$\sum m_{each\ object}\ v_{final\ y\ each\ object} - \sum m_{each\ object}\ v_{initial\ y\ each\ object} = 0$$
$$\sum W_{nc} = \sum \frac{1}{2} m_{each\ object}\ v^2_{final\ each\ object} - \sum \frac{1}{2} m_{each\ object}\ v^2_{initial\ each\ object}$$

Two Dimensional Elastic Collisions

During an elastic collision the two objects completely rebound off of each other as a result of the collision and there is no nonconservative work and no energy lost. The three equations that govern this situation are

$$\sum m_{each\ object}\ v_{final\ x\ each\ object} - \sum m_{each\ object}\ v_{initial\ x\ each\ object} = 0$$
$$\sum m_{each\ object}\ v_{final\ y\ each\ object} - \sum m_{each\ object}\ v_{initial\ y\ each\ object} = 0$$
$$0 = \sum \frac{1}{2} m_{each\ object}\ v^2_{final\ each\ object} - \sum \frac{1}{2} m_{each\ object}\ v^2_{initial\ each\ object}$$

The results of the three equations for this situation are

$$v_{1xf} = \frac{m_1 - m_2}{m_1 + m_2} v_{1xi} + \frac{2m_2}{m_1 + m_2} v_{2xi} \qquad v_{2xf} = \frac{2m_1}{m_1 + m_2} v_{1xi} + \frac{m_2 - m_1}{m_1 + m_2} v_{2xi}$$
$$v_{1yf} = v_{1yi} \qquad v_{2yf} = v_{2yi}$$

Angular Impulse and Momentum

Angular Momentum

Angular Momentum $\vec{L}$ is a vector value with a direction and is the measurement of the angular velocity around a center of mass $\vec{\omega}$ of an object or a system of objects, multiplied by the total moment of the object or a system of objects.

$$\vec{L}_{object} = I_{object}\ \vec{\omega}_{object}$$

$$\vec{L}_{system} = I_{system\ total}\ \vec{\omega}_{center\ of\ mass\ of\ the\ system}$$

$$\vec{L}_{point\ object} = \vec{r}_{from\ point\ or\ axis} \times \vec{p}_{object} = m_{object}\left(\vec{r}_{from\ point\ or\ axis} \times \vec{v}_{object}\right)$$

$$L_{point\ object} = m_{object}\ r_{from\ point\ or\ axis}\ v_{object}\ \sin\theta \quad where\ \theta\ is\ the\ tail\ to\ tail\ angle\ between\ \vec{r}\ and\ \vec{v}$$

Angular Impulse

Angular Impulse $\vec{A}$ is a vector value with a direction and is the action of a torque vector $\vec{\tau}$ times a time interval t during which that torque acts. The object does not need to rotate over the time interval for an impulse to be created.

$$\vec{A}_{system\ of\ forces} = \int \sum \vec{\tau}\ dt \quad = \sum \vec{\tau}\ t\ \ if\ the\ torque\ vector\ \vec{\tau}\ is\ constant$$

Angular Impulse Momentum Conservation

The total momentum vectors $\vec{L}$ of each object in a system of objects is only changed when there is a total impulse vector $\vec{A}$ acting on it, or equivalently when there is a nonzero net torque vector $\vec{\tau}$ acting on it through a time interval t

$$\vec{A}_{system\ of\ forces} = \sum \vec{L}_{final\ each\ object} - \sum \vec{L}_{initial\ each\ object} = \int \sum \vec{\tau}_{system\ of\ objects}\ dt$$

This can be expanded in the middle and written as

$$\vec{A}_{system\ of\ forces} = \sum I_{each\ object}\ \vec{\omega}_{final\ object} - \sum I_{each\ object}\ \vec{\omega}_{initial\ object} = \int \sum \vec{\tau}_{system\ of\ objects}\ dt$$

Gravitation

Force

The force of gravity is actually a variable Force that depends on the separation of the center of the two bodies

$$F = \frac{G\ m_1 m_2}{r^2}$$

Each body receives the same force but a different acceleration according to its mass.

$$G = 6.67 \times 10^{-11}\ N\ m^2/kg^2$$

$$r = radius\ of\ orbit$$

Surface Gravity

The gravitational acceleration experienced by one body on the surface of a second larger body of mass M and radius R

$$a_{surface} = g = \frac{G\ M}{R^2}$$

This Surface Gravitational Acceleration is only at the surface and will be less at other locations.

$$G = 6.67 \times 10^{-11}\ N\ m^2/kg^2$$

$$R = radius\ of\ larger\ body$$

Potential Energy

The potential energy of the conservative gravitational force is also variable and negative so that it increases the further apart the two objects are separated

$$U = -\frac{G\ m_1 m_2}{r}$$

Each body in the system has the same amount of potential energy.

$$r = R_{planet} + h_{orbit}$$

Work Energy Equation

The Work Energy equation in terms of the variable gravitational potential energy by the definition of Translational Kinetic Energy $K = \frac{1}{2}mv^2$ and Gravitational Potential Energy $U_g = -\frac{G\ m_1 m_2}{r}$ into becomes

$$\sum W_{nc} = \frac{1}{2}m\left(v_{final}^2 - v_{initial}^2\right) - G\ m_1 m_2\left(\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right)$$

Escape Speed

The speed needed for one body to escape the gravitational force of a second larger body of mass M and radius R

$$v_{escape} = \sqrt{\frac{2\ G\ M}{R}}$$

$$R = radius\ of\ larger\ body$$

Orbital Speed

The speed needed for one body to be in a stable circular orbit of radius r around a second larger body of mass M

$$v_{orbital} = \sqrt{\frac{G\ M}{r}}$$

$$r = radius\ of\ orbit$$

Keplers Third Law

Relation between period T and radius r of a stable circular orbit of one body around a second larger body of mass M

$$T^2 = \left(\frac{4\,\pi^2}{G\ M}\right) r^3$$

$$M = mass\ of\ larger\ body$$

$$r = radius\ of\ orbit$$

Simple Harmonic Motion

Simple Harmonic Motion is any kind of motion that repeats its exact transverse direction path over and over again in the same amount of time, this amount of time being known as the period T. Pendulums, waves, and circular paths are all examples of possible Simple Harmonic Motion. Simple Harmonic Motion has the following kinematic forms

Transverse Direction

Transverse Displacement
$x(t) = x_m \cos(\omega\, t + \phi)$
Transverse Velocity
$v(t) = \frac{dx(t)}{dt} = -\,\omega\, x_m\, \sin(\omega\, t + \phi)$
Transverse Acceleration
$a(t) = \frac{dv(t)}{dt} = -\omega^2\, x_m\, \cos(\omega\, t + \phi)$
Transverse Force
$F(t) = m\, a(t) = -m\, \omega^2\, x_m\, \cos(\omega\, t + \phi)$
$F(t) = -m\, \omega^2\, x(t) = -k\, x(t)$

Transverse Displacement
$x(t) = x_m \sin(\omega\, t + \phi)$
Transverse Velocity
$v(t) = \frac{dx(t)}{dt} = \ \omega\, x_m\, \cos(\omega\, t + \phi)$
Transverse Acceleration
$a(t) = \frac{dv(t)}{dt} = -\omega^2\, x_m\, \sin(\omega\, t + \phi)$
Transverse Force
$F(t) = m\, a(t) = -m\, \omega^2\, x_m\, \sin(\omega\, t + \phi)$
$F(t) = -m\, \omega^2\, x(t) = -k\, x(t)$

x_m is displacement amplitude or largest displacement ω is angular frequency or angular velocity t is time elapsed
m is mass in motion k is spring constant ϕ is phase angle in radians for sine and cosine combination $\tan\phi = -\frac{v(0)}{\omega\, x(0)}$

Maximum Transverse Displacement

$$x_m$$

Occurs with the following

$$displacement\ cos\ or\ sin = \pm 1$$
$$x(t) = max = +x_m\ \ or\ \ -x_m$$

or with the following

$$acceleration\ cos\ or\ sin = \pm 1$$
$$a(t) = max = +a_m\ \ or\ \ -a_m$$

or with the following

$$velocity\ sin\ or\ cos = 0$$
$$v(t) = 0$$

Maximum Transverse Velocity

$$v_m = \omega\, x_m$$

Occurs with the following

$$velocity\ cos\ or\ sin = \pm 1$$
$$v(t) = max = +v_m\ \ or\ \ -v_m$$

or with the following

$$displacement\ cos\ or\ sin = 0$$
$$y(t) = 0$$

or with the following

$$acceleration\ cos\ or\ sin = 0$$
$$a(t) = 0$$

Maximum Transverse Acceleration

$$a_m = \omega\, v_m = \omega^2\, x_m$$

Occurs with the following

$$acceleration\ cos\ or\ sin = \pm 1$$
$$a(t) = max = +a_m\ \ or\ \ -a_m$$

or with the following

$$displacement\ cos\ or\ sin = \pm 1$$
$$x(t) = max = +x_m\ \ or\ \ -x_m$$

or with the following

$$velocity\ sin\ or\ cos = 0$$
$$v(t) = 0$$

Simple Harmonic Motion Relations

Natural Angular Frequency

$$\omega = 2\,\pi\, f = \frac{2\,\pi}{T}$$
$$\omega = \sqrt{\frac{k}{m}}$$

Natural Cycle Frequency

$$f = \frac{\omega}{2\,\pi} = \frac{1}{T}$$

Natural Cycle Period

$$T = \frac{2\,\pi}{\omega} = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

Oscillation Constant

$$k = -\frac{F(t)}{x(t)} = -\frac{m\, a(t)}{x(t)}$$
$$k = \frac{m\, a_m}{x_m} = m\, \omega^2$$

Resonance occurs in a simple harmonic system when the external drive frequency ω_d matches the natural frequency ω

$$\omega_d = \omega \qquad Resonance$$

Energy Relations

A simple harmonic oscillator has kinetic energy $K(t)$, potential energy $U(t)$, and total mechanical energy $E(t)$ functions of time. The total mechanical energy function of time $E(t)$ is a constant value throughout the motion.

$$K(t) = \frac{1}{2}\, m\, v(t)^2 = \frac{1}{2}\, m\, \omega^2\, x_m^2\, \sin^2(\omega\, t + \phi) = \frac{1}{2}\, k\, x_m^2\, \sin^2(\omega\, t + \phi)$$
$$U(t) = \frac{1}{2}\, k\, x(t)^2 = \frac{1}{2}\, m\, \omega^2\, x_m^2\, \cos^2(\omega\, t + \phi) = \frac{1}{2}\, k\, x_m^2\, \cos^2(\omega\, t + \phi)$$

$$E(t) = K(t) + U(t) = \frac{1}{2}\, k\, x_m^2\, \sin^2(\omega\, t + \phi) + \frac{1}{2}\, k\, x_m^2\, \cos^2(\omega\, t + \phi) = \frac{1}{2}\, k\, x_m^2 = \frac{1}{2}\, m\, v_m^2 = \frac{1}{2}\, k\, x^2 + \frac{1}{2}\, m\, v^2$$

Simple Harmonic Oscillator

A simple harmonic oscillator is any system that has a restoring force that is directly proportional to the distance to the first power. A spring mass system or any spring like system behaves as a simple harmonic oscillator. If the displacement of the system is small, it will undergo Simple Harmonic Motion with an angular frequency ω and a period T given by

$$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{\omega}{2\,\pi} = \frac{1}{T} = \frac{1}{2\,\pi}\sqrt{\frac{k}{m}} \qquad T = \frac{2\,\pi}{\omega} = \frac{1}{f} = 2\,\pi\sqrt{\frac{m}{k}}$$

m is the mass that is connected on the end of a spring or the spring-like system
k is the force constant, the proportionality constant between the position and the restoring force $F = -k\,x$

Torsional Pendulum

If the pendulum system involves an object on a long string or rope of negligible mass, it can undergo Oscillatory Motion as a Torsional Pendulum. If the string or rope that connects the object to the ceiling is twisted to only a small angle with the vertical, it will undergo Simple Harmonic Motion with an angular frequency ω and a period T given by

$$\omega = \sqrt{\frac{\kappa}{I}} \qquad f = \frac{\omega}{2\,\pi} = \frac{1}{T} = \frac{1}{2\,\pi}\sqrt{\frac{\kappa}{I}} \qquad T = \frac{2\,\pi}{\omega} = \frac{1}{f} = 2\,\pi\sqrt{\frac{I}{\kappa}}$$

I is the total angular inertia or total moment of inertia of the object or system around the pivot axis
κ is the torque constant, the proportionality constant between the angular position and the restoring torque $\tau = -\kappa\,\theta$

Physical Pendulum

If an object or a system is hinged at a point other than its own center of mass, it can undergo Oscillatory Motion as a Physical Pendulum if the hinge point and the center of mass lie at different vertical heights. If the line that connects the two points forms only a small angle with the vertical, it will undergo Simple Harmonic Motion with an angular frequency ω and a period T given by

$$\omega = \sqrt{\frac{m\,g\,h}{I}} \qquad f = \frac{\omega}{2\,\pi} = \frac{1}{T} = \frac{1}{2\,\pi}\sqrt{\frac{m\,g\,h}{I}} \qquad T = \frac{2\,\pi}{\omega} = \frac{1}{f} = 2\,\pi\sqrt{\frac{I}{m\,g\,h}}$$

I is the total angular inertia or total moment of inertia of the object or system around the pivot axis
m is the total linear inertia or total mass of the object or system
g is the gravitational acceleration, at the earth's surface $g = 9.81\ \frac{m}{s^2}$
h is the distance between the center of mass of the object or system and the axis of rotation

$$h = \frac{\sum m_n\,h_n}{\sum m_n}$$

Simple Pendulum

If the physical pendulum system involves an object on a long string or rope of negligible mass, it can undergo Oscillatory Motion as a Simple Pendulum. If the line that connects the two points forms only a small angle with the vertical, it will undergo Simple Harmonic Motion with a period T that simplifies to and is given by

$$\omega = \sqrt{\frac{g}{L}} \qquad f = \frac{\omega}{2\,\pi} = \frac{1}{T} = \frac{1}{2\,\pi}\sqrt{\frac{g}{L}} \qquad T = \frac{2\,\pi}{\omega} = \frac{1}{f} = 2\,\pi\sqrt{\frac{L}{g}}$$

L is the length of the string or rope which must be much longer than any dimension of the object
g is the gravitational acceleration, at the earth's surface $g = 9.81\ \frac{m}{s^2}$

Damped Harmonic Motion

Damped Harmonic Motion is any kind of motion that repeats its exact transverse direction path over and over again in the same amount of time, this amount of time being known as the period T, but each successive amplitude of the motion is lesser than the previous amplitude value due to nonconservative work done on the harmonic oscillator.

Transverse Direction

Transverse Displacement

$x(t) = x_m\, e^{-\frac{b\,t}{2\,m}} \cos(\omega'\,t + \phi)$

Transverse Velocity

$v(t) = \frac{dx(t)}{dt} = -\,\omega'\,x_m\, e^{-\frac{b\,t}{2\,m}} \sin(\omega'\,t + \phi)$

Transverse Acceleration

$a(t) = \frac{dv(t)}{dt} = -\omega'^2\,x_m\, e^{-\frac{b\,t}{2\,m}} \cos(\omega'\,t + \phi)$

Transverse Force

$F(t) = m\,a(t) = -m\,\omega'^2\,x_m\, e^{-\frac{b\,t}{2\,m}} \cos(\omega'\,t + \phi)$

$F(t) = -m\,\omega'^2\,x(t) = -k\,x(t)$

Transverse Displacement

$x(t) = x_m\, e^{-\frac{b\,t}{2\,m}} \cos(\omega'\,t + \phi)$

Transverse Velocity

$v(t) = \frac{dx(t)}{dt} = -\,\omega'\,x_m\, e^{-\frac{b\,t}{2\,m}} \sin(\omega'\,t + \phi)$

Transverse Acceleration

$a(t) = \frac{dv(t)}{dt} = -\omega'^2\,x_m\, e^{-\frac{b\,t}{2\,m}} \cos(\omega'\,t + \phi)$

Transverse Force

$F(t) = m\,a(t) = -m\,\omega'^2\,x_m\, e^{-\frac{b\,t}{2\,m}} \cos(\omega'\,t + \phi)$

$F(t) = -m\,\omega'^2\,x(t) = -k\,x(t)$

x_m is displacement amplitude or largest displacement ω' is angular frequency or angular velocity m is mass in motion
k is spring constant b is damping constant t is time elapsed ϕ is phase angle in radians $\tan\phi = -\frac{v(0)}{\omega\,x(0)}$

Maximum Transverse Displacement

x_m

Occurs with the following

$displacement\ cos\ or\ sin = \pm 1$

$x(t) = max = +x_m\ \ or\ \ -x_m$

or with the following

$acceleration\ cos\ or\ sin = \pm 1$

$a(t) = max = +a_m\ \ or\ \ -a_m$

or with the following

$velocity\ sin\ or\ cos = 0$

$v(t) = 0$

Maximum Transverse Velocity

$v_m = \omega\,x_m$

Occurs with the following

$velocity\ cos\ or\ sin = \pm 1$

$v(t) = max = +v_m\ \ or\ \ -v_m$

or with the following

$displacement\ cos\ or\ sin = 0$

$y(t) = 0$

or with the following

$acceleration\ cos\ or\ sin = 0$

$a(t) = 0$

Maximum Transverse Acceleration

$a_m = \omega\,v_m = \omega^2\,x_m$

Occurs with the following

$acceleration\ cos\ or\ sin = \pm 1$

$a(t) = max = +a_m\ \ or\ \ -a_m$

or with the following

$displacement\ cos\ or\ sin = \pm 1$

$x(t) = max = +x_m\ \ or\ \ -x_m$

or with the following

$velocity\ sin\ or\ cos = 0$

$v(t) = 0$

Simple Harmonic Motion Relations

Natural Angular Frequency

$\omega' = 2\,\pi\,f' = \frac{2\,\pi}{T'}$

$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4\,m^2}} \approx \sqrt{\frac{k}{m}}$

Natural Cycle Frequency

$f' = \frac{\omega'}{2\,\pi} = \frac{1}{T'}$

Natural Cycle Period

$T' = \frac{2\,\pi}{\omega'} = \frac{1}{f'} \approx 2\pi\sqrt{\frac{m}{k}}$

Oscillation Constant

$k = -\frac{F(t)}{x(t)} = -\frac{m\,a(t)}{x(t)} = \frac{m\,a_m}{x_m}$

$k = m\,\omega'^2$

Energy Relations

A simple harmonic oscillator has kinetic energy $K(t)$, potential energy $U(t)$, and total mechanical energy $E(t)$ functions of time. The total mechanical energy function of time $E(t)$ is an exponentially decreasing value throughout the motion.

$$K(t) = \frac{1}{2}\,m\,v(t)^2 = \frac{1}{2}\,m\,\omega'^2\,x_m^2\,e^{-\frac{b\,t}{m}} \sin^2(\omega'\,t + \phi) = \frac{1}{2}\,k\,x_m^2\,e^{-\frac{b\,t}{m}} \sin^2(\omega'\,t + \phi)$$

$$U(t) = \frac{1}{2}\,k\,x(t)^2 = \frac{1}{2}\,m\,\omega'^2\,x_m^2\,e^{-\frac{b\,t}{m}} \cos^2(\omega'\,t + \phi) = \frac{1}{2}\,k\,x_m^2\,e^{-\frac{b\,t}{m}} \cos^2(\omega'\,t + \phi)$$

$$E(t) = K(t) + U(t) = \frac{1}{2}\,k\,x_m^2\,e^{-\frac{b\,t}{m}} \sin^2(\omega'\,t + \phi) + \frac{1}{2}\,k\,x_m^2\,e^{-\frac{b\,t}{m}} \cos^2(\omega'\,t + \phi) = \frac{1}{2}\,k\,x_m^2\,e^{-\frac{b\,t}{m}} = E_m\,e^{-\frac{b\,t}{m}}$$

Wave Motion

Wave Motion is any kind of motion that repeats its exact transverse direction path over and over again at a fixed point of space in the same amount of time, this amount of time being known as the period T, while an oscillation between maximum and minimum values continues to move across the point in the longitudinal direction. Fluids, stretched strings, sound, and light are all examples of possible Wave Motion. Wave Motion has the following kinematic forms

Transverse Direction Relations

The Transverse Direction is the actual direction for the oscillation of mass particles as a wave front passes through.

Transverse Displacement

$y(x,t) = y_m \cos(k\,x \pm \omega\,t + \phi)$

$\pm$ will be positive for leftward moving waves

$\pm$ will be negative for rightward moving waves

Transverse Velocity

$v(x,t) = \frac{\partial y(x,t)}{\partial t} = \mp\,\omega\,y_m \sin(k\,x \pm \omega\,t + \phi)$

Transverse Acceleration

$a(x,t) = \frac{\partial v(x,t)}{\partial t} = -\omega^2\,y_m \cos(k\,x \pm \omega\,t + \phi)$

$a(x,t) = -\,\omega^2\,y(x,t)$

Transverse Force

$F(x,t) = -m\,\omega^2\,y_m \cos(k\,x \pm \omega\,t + \phi)$

$F(x,t) = -m\,\omega^2\,y(x,t)$

Transverse Displacement

$y(x,t) = y_m \sin(k\,x \pm \omega\,t + \phi)$

$\pm$ will be positive for leftward moving waves

$\pm$ will be negative for rightward moving waves

Transverse Velocity

$v(x,t) = \frac{\partial y(x,t)}{\partial t} = \pm\,\omega\,y_m \cos(k\,x \pm \omega\,t + \phi)$

Transverse Acceleration

$a(x,t) = \frac{\partial v(x,t)}{\partial t} = -\omega^2\,y_m \sin(k\,x \pm \omega\,t + \phi)$

$a(x,t) = -\,\omega^2\,y(x,t)$

Transverse Force

$F(x,t) = -m\,\omega^2\,y_m \sin(k\,x \pm \omega\,t + \phi)$

$F(x,t) = -m\,\omega^2\,y(x,t)$

y_m is displacement amplitude or the largest displacement x is fixed position along the wave front t is time elapsed

k is angular wave number ω is angular frequency or angular velocity ϕ is phase angle in radians $\tan\phi = -\frac{v(0,0)}{\omega\,y(0,0)}$

Maximum Transverse Displacement

$$y_m$$

Occurs with the following

$$displacement\ cos\ or\ sin = \pm 1$$
$$y(x,t) = max = +y_m\ \ or\ -y_m$$

or with the following

$$acceleration\ cos\ or\ sin = \pm 1$$
$$a(x,t) = max = +a_m\ \ or\ -a_m$$

or with the following

$$velocity\ sin\ or\ cos = 0$$
$$v(x,t) = 0$$

Maximum Transverse Velocity

$$v_m = \omega\,y_m$$

Occurs with the following

$$velocity\ cos\ or\ sin = \pm 1$$
$$v(x,t) = max = +v_m\ \ or\ -v_m$$

or with the following

$$displacement\ cos\ or\ sin = 0$$
$$y(x,t) = 0$$

or with the following

$$acceleration\ cos\ or\ sin = 0$$
$$a(x,t) = 0$$

Maximum Transverse Acceleration

$$a_m = \omega\,v_m = \omega^2\,y_m$$

Occurs with the following

$$acceleration\ cos\ or\ sin = \pm 1$$
$$a(x,t) = max = +a_m\ \ or\ -a_m$$

or with the following

$$displacement\ cos\ or\ sin = \pm 1$$
$$y(x,t) = max = +y_m\ \ or\ -y_m$$

or with the following

$$velocity\ sin\ or\ cos = 0$$
$$v(x,t) = 0$$

Longitudinal Direction Relations

The Longitudinal Direction is perpendicular to the Tranverse Direction and is the direction of wave propagation which has a wave speed v related to its angular frequency ω, wave number k,wavelength λ, period T, and cycle frequency f.

$$Longitudinal\ Wave\ Propagation\ Speed \qquad v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda\,f$$

Wave Motion Relations

Wave relations for wave speed v, angular frequency ω, wave number k,wavelength λ, period T, and cycle frequency f.

Natural Angular Frequency

$$\omega = 2\,\pi\,f = \frac{2\pi}{T}$$

Natural Cycle Frequency

$$f = \frac{\omega}{2\,\pi} = \frac{1}{T}$$

Natural Cycle Period

$$T = \frac{2\,\pi}{\omega} = \frac{1}{f}$$

Wave Number

$$k = \frac{2\,\pi}{\lambda}$$

Wavelength

$$\lambda = \frac{2\,\pi}{k}$$

$$\lambda = \frac{v}{f}$$

Phase Difference, Location Difference, Time Difference

The Phase Difference is the angular difference in radians between two points that differ in location or in time on a wave.

Location Difference

$\Delta x = \frac{\Delta\phi}{2\pi}\lambda$

Δx Location difference between two points on a wave

$\Delta\phi$ Phase difference between two points on a wave

λ Wavelength of the wave

Time Difference

$\Delta t = \frac{\Delta\phi}{2\pi}T$

Δt Time difference between two points on a wave

$\Delta\phi$ Phase difference between two points on a wave

T Period of the wave

Standing Waves

Standing Wave motion results when two wave fronts with matching amplitude, angular frequency, and angular wave number but moving in opposite directions interfere with each other and combine to form a new wave front form. The wave front form in this situation has the time and space components of the equation in separate trig functions.

$$y(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) = 2\, y_m \sin(kx)\cos(\omega t)$$

$$v(x,t) = \frac{\partial y(x,t)}{\partial t} = -2\,\omega\, y_m \sin(kx)\sin(\omega t) \qquad a(x,t) = \frac{\partial v(x,t)}{\partial t} = -2\,\omega^2\, y_m \sin(kx)\cos(\omega t)$$

Due to this separation the nodes remain in certain locations as time progresses and the wave appears to remain stationary. The location of the nodes (zeros) and the antinodes (maxima) are only at certain allowed values

Allowed Node (Zero) Location, Wavelength, Frequency

$x = \frac{n\,\lambda}{2}$ $\quad \lambda = \frac{2L}{n}$ $\quad f = \frac{n\,v}{2\,L}$

Allowed Antinode (Max) Location, Wavelength, Frequency

$x = \frac{(2n-1)\,\lambda}{4}$ $\quad \lambda = \frac{4L}{(2n-1)}$ $\quad f = \frac{(2n-1)v}{4\,L}$

n is any positive integer: $n = 1,2,3,4,\ldots$

The maximum amplitude achieved by the antinode locations is $2y_m$

Stretched String Waves

Waves that travel on a stretched string with both ends clamped have a speed related to the Tension T in the string

$$v = \sqrt{\frac{T}{\mu}}$$

T is the Tension in the string, which is either known or can be found from Newtons Second Law

μ is the Linear Mass Density of the string, which is the mass per unit length $\mu = \frac{m}{L}$

Power or Rate at which the energy is transmitted by the stretched string $P_{avg} = \frac{1}{2}\,\mu\, v\, \omega^2\, y_m^2$

Waves that travel on a stretched string with both ends clamped are in Standing Wave Motion, with an extra condition that both ends of the string are nodes, so that only certain wavelengths and frequencies of waves are allowed. The longest allowed wavelength corresponds to the lowest allowed frequency and is known as the fundamental frequency. The wavelength of the fundamental frequency is exactly twice the length of the string. All allowed values are

Allowed Wavelengths

$\lambda = \frac{2L}{n}$

Allowed Frequencies

$f = \frac{v}{\lambda} = \frac{nv}{2L}$

n is any positive integer: $n = 1,2,3,4,\ldots$

The fundamental frequency and the fundamental wavelength correspond to $n = 1$

Interference of Waves

Interference of Wave motion results when two wave fronts with matching amplitude, angular frequency, and angular wave number but phase shifted interfere with each other and combine to form a new wave front form. The wave front form in this situation has the time and space components of the equation in separate trig functions.

$$y(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t + \phi) = 2\, y_m \cos\left(\frac{1}{2}\phi\right)\sin\left(kx + \omega t + \frac{1}{2}\phi\right)$$

Due to this phase shift ϕ the resultant wave amplitude during the Interference of Waves is phase shift ϕ dependent

$$amplitude = 2\, y_m \cos\left(\frac{1}{2}\phi\right)$$